

Chapter 7 Integration

Basic methods of Integration (Sections 7.1 & 7.2)

- *Guess and Check*: Use intuition, be bold in guessing but check carefully by differentiation.
- *Substitution*: If you can recognize a function $w(x)$ such that the integrand is of the form $f(w(x))w'(x)$ where f is a simple function whose antiderivative you know, then you may use substitution.
- *Integration by parts*: Let u, v be two function of x , then

$$\int uv' dx = uv - \int u' v dx$$

Study (don't just read through but work through) all examples in Sections 7.1 & 7.2)!!!

Partial Fractions and Trigonometric Substitutions (Section 7.4)

- *The method of partial fractions* is useful for integrating rational functions of the form $P(x)/Q(x)$ where P and Q are polynomials. Refer to box on P.334 of text for a good strategy of applying partial fractions.
- *The method of trigonometric substitutions* (usually substituting x by $\tan \theta$ or $\sin \theta$) is useful for integrating functions which involve powers of quadratics of the form $a^2 \pm x^2$.

Special Announcements on 9/5

1. Due to a meeting, Dr. Leung's office hours today (Tues. 9/5) is moved to 4:30-6 p.m.
2. **Class on Thursday 9/7 (ONLY!) will be held in LART 222.** We'll meet at LART 403 on all other class days.
3. TA for this class:
Bereket Weldeslassie
Bell Hall 206, x6756
Email: btweldeslassie@utep.edu
Office Hours: MF 11:30 a.m. – 12:30 p.m.
4. Question for class: Is it OK to post your nice works on web?

Definite Integral

If $\int f(x)dx = F(x) + C$, then the definite integral of $f(x)$ between the real numbers a and b with is

$$\int_a^b f(x)dx = F(b) - F(a)$$

Improper Integral (Section 7.7)

A definite integral $\int_a^b f(x)dx$ is called improper if

- (a) the upper or lower (or both) limits of integration is infinity; or
- (b) the integrand $f(x)$ has infinite value for some x between a and b .

Improper integrals should be calculated as a **limit**. If the limit exists and turns out to be a finite number, we say that the improper integral converges, otherwise we say that it diverges.

Announcements on 9/7 and 9/12

1. Dr. Leung's office hours this semester will be changed to W 12-1:30 p.m. and R 1:30 – 3 p.m.
2. Free tutoring services available at the Tutoring and Learning Center. Calc. II Study Group every Friday 9-11 a.m.
3. Club Zero meeting on Wed. 9/13 in Bell Hall, room 143.
4. Reminder: First midterm test on Thursday 9/21.

Comparison of Improper Integrals (Section 7.8)

Sometimes it is difficult to find the exact value of an improper integral by antidifferentiation.

However, it may still be possible to determine whether an integral converges or diverges by comparing it with one whose behavior we already know.

The Comparison Test for $\int_a^\infty f(x)dx$ (p.357)

Assume $f(x)$ is positive. Making a comparison involves two steps:

1. Look at the behavior of the integrand for large values of x . Guess whether the integral converges or not.
2. Confirm the guess by comparison:
 - If $0 \leq f(x) \leq g(x)$ and $\int_a^\infty g(x)dx$ converges, then $\int_a^\infty f(x)dx$ converges
 - If $0 \leq g(x) \leq f(x)$ and $\int_a^\infty g(x)dx$ diverges, then $\int_a^\infty f(x)dx$ diverges

Standard integrals useful for comparison:

- $\int_1^{\infty} \frac{1}{x^p} dx$ converges for $p > 1$,
diverges for $p \leq 1$.
- $\int_0^1 \frac{1}{x^p} dx$ converges for $p < 1$,
diverges for $p \geq 1$.
- $\int_0^{\infty} e^{-ax} dx$ converges for $a > 0$.

Study Examples 1, 3, 4, and 5 in Section 7.8

Area as a Definite Integral

The area bounded by the curve $y = f(x)$, the horizontal axis, and the vertical lines $x = a$ and $x = b$ is given by the definite integral $\int_a^b f(x)dx$.

Note that the definite integral gives a signed area. That is, if f is positive (resp. negative) over the range of integration, the area calculated is positive (resp. negative). If f changes sign once or more over the range of integration, the positive and negative areas might cancel out one another. When calculating the absolute area, it is very important to sketch the graph of f and know exactly where the function changes sign.

Approximating Definite Integrals

In many cases, it is not easy to work out a definite integral by antidifferentiation. We may still get an approximate the integral numerically. The idea is to cut up the area under the curve into thin slices and approximate the areas of the slices by rectangles or trapezoids. These approximate methods may underestimate or overestimate the integral. The thinner the slices are, the smaller the approximation error is.

Popular approximation methods include the Left rule, Right rule, Midpoint rule, Trapezoid rule, and Simpson's rule (see Sections 7.5 and 7.6)