$\frac{19}{20}$

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36. $\int \frac{dz}{z^2+z} = \int \frac{dz}{z(z+1)} = \frac{A}{z} + \frac{B}{z+1}$

$dz = 1$

$1 = A(z+1) + B(z)$

$1 = Az + A + Bz$

$1 = (A+B)z + A$

$A+B=0$

$(A=1)-1$

$-A = -1$

$B=-1 \quad A=1$

$\int \frac{1}{z} + \frac{-1}{z+1} = \ln|z| - \ln|z+1| = \boxed{\ln\left|\frac{z}{z+1}\right| + C}$ ✓

38. $\int \frac{dP}{3P-3P^2} = \int \frac{dP}{-3P(P-1)} = \frac{A}{-3P} + \frac{B}{P-1}$

$dP = 1$

$1 = A(P-1) + B(-3P)$

$1 = AP - A + -3BP$

$1 = (A-3B)P - A$

$A - 3B = 0$

$-A = 1 \quad A = -1$

$-3B = 1 \quad B = -\frac{1}{3}$

$B = -\frac{1}{3}$

$\frac{-1}{-3P} + \frac{-1/3}{P-1}$

$\frac{1}{3P} + \frac{-1/3}{P-1}$

$\frac{1/3}{P} - \frac{1/3}{P-1}$

$\int \frac{1/3}{P} - \frac{1/3}{P-1} = \frac{1}{3}\ln|P| - \frac{1}{3}\ln|P-1|$

$\boxed{\frac{1}{3}\ln\left|\frac{P}{P-1}\right| + C}$ ✓

by substitution

56. $\int \frac{\cos}{1-\sin^2}$ $\qquad w = \sin$

$\qquad dw = \cos$

$$\int \frac{dw}{1-w^2} = \int dw \cdot \frac{1}{1-w^2} = \int dw \cdot \frac{1}{-(w-1)(w+1)} =$$

$$\frac{1}{-(w-1)(w+1)} = \frac{A}{-(w-1)} + \frac{B}{(w+1)}$$

$1 = A(w+1) + B(-w+1)$

$1 = Aw + A + -Bw + B$

$1 = (A + -B)w + (A+B)$

$A - \cancel{B} = 0$

$A + \cancel{B} = 1$

$2A = 1$

$A = 1/2 \quad B = 1/2$

$$\frac{1/2}{-(w-1)} + \frac{1/2}{(w+1)} = \frac{1/2}{w+1} - \frac{1/2}{w-1} = \int dw \cdot \frac{1/2}{w+1} - \frac{1/2}{w-1} =$$

$$1/2 \ln|w+1| - 1/2|w-1| = 1/2 \ln\left|\frac{w+1}{w-1}\right| =$$

$$\boxed{1/2 \ln\left(\left|\frac{\sin(x)+1}{\sin(x)-1}\right|\right) + C}$$

by trig sub. on next pg.

56. by trig sub.

$$\int \frac{dx}{1-x^2} \qquad x = \sin\theta \qquad dx = \cos\theta \, d\theta$$

$$\int \frac{\cos\theta}{1-\sin\theta^2}\, d\theta = \int \frac{\cos\theta}{\cos\theta^2}\, d\theta = \int \frac{1}{\cos\theta}\, d\theta =$$

$$\boxed{\tfrac{1}{2} \ln \left| \frac{\sin\theta + 1}{\sin\theta - 1} \right| + C} \leftarrow \text{Identity}$$

62. $a > 0 \qquad \int \frac{1}{x^2 - a}\, dx \qquad$ a is positive

$$\int \frac{1}{x^2-a} = \int \frac{1}{(x-\sqrt{a})(x+\sqrt{a})} = \frac{A}{(x-\sqrt{a})} + \frac{B}{(x+\sqrt{a})}$$

$$A(x+\sqrt{a}) + B(x-\sqrt{a}) = Ax + A\sqrt{a} + Bx - B\sqrt{a} =$$

$$(A+B)x + (A-B)\sqrt{a} = 1$$

$$A + B = 0$$

$$\sqrt{a}(A - B) = 1$$

$$2A\sqrt{a} = 1$$

$$A = 1/2\sqrt{a} \qquad B = -1/2\sqrt{a}$$

$$\int \frac{1/2\sqrt{a}}{(x-\sqrt{a})} + \frac{-1/2\sqrt{a}}{(x+\sqrt{a})} = 1/2\sqrt{a} \ln \left| \frac{x-\sqrt{a}}{x+\sqrt{a}} \right| =$$

$$\boxed{\frac{1 \ \ln \left| \frac{x-\sqrt{a}}{x+\sqrt{a}} \right|}{2\sqrt{a}}}$$

$a=0$ $\int \frac{1}{x^2-0} = \int \frac{1}{x^2}$

$$\int \frac{1}{x^2} = \int x^{-2} = \int -x^{-1} = \frac{1}{-x} = \boxed{-\frac{1}{x}+c}$$

$a<0$ $\int \frac{1}{x^2+a}$ $\sqrt{a}=b$ Identity #24

$$\int \frac{1}{x^2+a} = \int \frac{1}{x^2+b^2} = \frac{1}{b}\arctan\left(\frac{x}{b}\right)+C =$$

$$\frac{1}{\sqrt{a}}\arctan\left(\frac{x}{\sqrt{a}}\right) = \boxed{\frac{\arctan\left(\frac{x}{\sqrt{a}}\right)}{\sqrt{a}}+C}$$