Homework

19/20

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36. $\int \frac{dz}{z^2+z} = \int \frac{1}{z(z+1)}\,dz$

$\left(\frac{1}{z(z+1)} = \frac{A}{z} + \frac{B}{z+1}\right) z(z+1)$

$1 = A(z+1) + Bz$

$1 = Az + A + Bz$

$A - 1 = 0 \qquad Az + Bz = 0$

$A = 1 \qquad z(A+B) = 0$

$A + B = 0$

$B = -1$

$\int \frac{dz}{z^2+z} = \ln|z| - \ln|z+1| + C$ ✓

38. $\int \frac{dp}{3p-3p^2} = \int \frac{dp}{3p(1-p)} = \frac{1}{3}\int \frac{1}{p(1-p)}\,dp$

$\left(\frac{1}{p(1-p)} = \frac{A}{p} + \frac{B}{1-p}\right) p(1-p)$

$1 = A(1-p) + Bp$

$1 = A - Ap + Bp$

$A = 1 \qquad -p(A-B) = 0$

$A - B = 0$

$1 - B = 0$

$B = 1$

$\int \frac{dp}{3p-3p^2} = \frac{1}{3}\left(\ln|p| + \ln|1-p|\right) + C$ ✓

56. $\int \frac{dx}{1-x^2}$ $x = \sin\theta$

$dx = \cos\theta$ $\theta = \arcsin x$

$\int \frac{dx}{(1+x)(1-x)}$

$\int \frac{\cos\theta}{1-\sin^2\theta} \int \frac{\cos\theta}{\cos^2\theta} = \int \frac{1}{\cos\theta}$

$(1+x)(1-x)\left(\frac{1}{(1+x)(1-x)} = \frac{A}{1+x} + \frac{B}{1-x}\right)$

$= \frac{1}{2}\ln\left|\frac{\sin\theta+1}{\sin\theta-1}\right| + C$ ✓

$= \frac{1}{2}\ln\left|\frac{\sin(\arcsin x)+1}{\sin(\arcsin x)-1}\right| + C$

$= \boxed{\frac{1}{2}\ln\left|\frac{x+1}{x-1}\right| + C}$

$1 = A(1-x) + B(1+x)$

$1 = A - Ax + B + Bx$

$A + B = 1 \quad -Ax + Bx = 0$

$x(-A+B) = 0$

$-A + B = 0$

$A + B = 1$
$-A + B = 0$
$2B = 1$ $\quad B = 1/2$ $\quad A = 1/2$

$= \frac{1}{2}(\ln|1+x| + \ln|1-x|) + C$

$\frac{1}{2}(\ln|1+x| - \ln|x-1|)$

$\boxed{\frac{1}{2}\ln\left(\frac{x+1}{x-1}\right) + C}$ ✓

62. $\int \frac{1}{x^2-a}\,dx$

a) $a > 0$

$\int \frac{1}{x^2-a} = -\int \frac{1}{a-x^2}$

$(\sqrt{a}+x)(\sqrt{a}-x)\left(\frac{1}{(\sqrt{a}+x)(\sqrt{a}-x)} = \frac{A}{\sqrt{a}+x} + \frac{B}{\sqrt{a}-x}\right)$

$1 = A(\sqrt{a}-x) + B(\sqrt{a}+x)$

$1 = A\sqrt{a} - Ax + B\sqrt{a} + Bx$

$-Ax + Bx = 0$

$(-A+B=0)\sqrt{a}$

$\sqrt{a}A + \sqrt{a}B = 1$

$-\sqrt{a}A + \sqrt{a}B = 0$

$2\sqrt{a}B = 1 \rightarrow B = \frac{1}{2\sqrt{a}}$

$A = \frac{1}{2\sqrt{a}}$

$-\int$

$\frac{1}{2\sqrt{a}}\ln|\sqrt{a}+x| + \frac{1}{2\sqrt{a}}\ln|\sqrt{a}-x| + C$

$\boxed{\frac{-1}{2\sqrt{a}}(\ln|\sqrt{a}+x| + \ln|\sqrt{a}-x|) + C}$ ✓

b) $a = 0$

$\int \frac{1}{x^2}\,dx = \int x^{-2}\,dx$

$\frac{x^{-1}}{-1} = \boxed{-\frac{1}{x} + C}$ ✓

c) $a < 0$

$\int \frac{1}{x^2+a}$ $\quad \theta = \arctan\frac{x}{\sqrt{a}}$

$x = \sqrt{a}\tan\theta$ $\quad dx = \frac{\sqrt{a}}{\cos^2\theta}$

$\int \frac{1}{(\sqrt{a}\tan\theta)^2 + a} \cdot \frac{\sqrt{a}}{\cos^2\theta}$

$\int \frac{1}{a\tan^2\theta + a} \cdot \frac{\sqrt{a}}{\cos^2\theta}$

$\int \frac{1}{a(\tan^2\theta + 1)} \cdot \frac{\sqrt{a}}{\cos^2\theta}$

$\frac{\sqrt{a}}{a}\int \frac{1}{\frac{\sin^2\theta}{\cos^2\theta} + 1} \cdot \frac{1}{\cos^2\theta}$

$\frac{\sqrt{a}}{a}\int \frac{1}{\sin^2\theta + \cos^2\theta} \cdot \frac{1}{\cos^2\theta}$

$\frac{\sqrt{a}}{a}\int \frac{1}{\frac{1}{\cos^2\theta}} \cdot \frac{1}{\cos^2\theta} = \frac{1}{\sqrt{a}}\int 1\,d\theta$ ✓

$= \frac{1}{\sqrt{a}}\theta = \frac{1}{\sqrt{a}}\arctan\frac{x}{\sqrt{a}} + C$