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$\frac{20}{20}$

HOMEWORK p.355

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(10.) $\int_1^\infty \frac{x}{4+x^2}\,dx$

$f(\omega) = \frac{1}{\omega}$

$\omega = 4+x^2$

$d\omega = 2x$

$\frac{1}{2}\int \frac{1}{\omega}\,d\omega = \frac{1}{2}\ln\omega = \frac{1}{2}\ln(4+x^2)$

$\int_1^\infty \frac{x}{4+x^2} = \lim_{b\to\infty}\int_1^b \frac{x}{4+x^2}\,dx = \frac{1}{2}\ln(4+x^2)\Big|_1^b = \frac{1}{2}\ln(4+b^2) - \frac{1}{2}\ln(5)$

$= \frac{1}{2}(\ln(4+b^2) - \ln 5) = \frac{1}{2}\ln\left(\frac{4+b^2}{5}\right)$

5 $\int_1^\infty \frac{x}{4+x^2}\,dx = \lim_{b\to\infty}\int_1^b \frac{x}{4+x^2}\,dx = \lim_{b\to\infty}\left(\frac{1}{2}\ln\frac{4+b^2}{5}\right) = \infty$ ✓

$\int_1^\infty \frac{x}{4+x^2}\,dx \Rightarrow$ diverges

(26) $\int_3^\infty \frac{dx}{x(\ln x)^2}$ $\quad f(\omega) = \frac{1}{\omega^2} = \omega^{-2}$

$\omega = \ln x$

$d\omega = 1/x$

5 $\int \omega^{-2}\,d\omega = \frac{\omega^{-1}}{-1} = -\frac{1}{\omega}$ $\quad \int \frac{dx}{x(\ln x)^2} = -\frac{1}{\ln x}$ ✓

$\int_3^\infty \frac{dx}{x(\ln x)^2} = \lim_{b\to\infty}\int_3^b \frac{dx}{x(\ln x)^2} = \frac{-1}{\ln x}\Big|_3^b = \frac{-1}{\ln b} - \left(\frac{-1}{\ln 3}\right) = \frac{-1}{\ln b} + \frac{1}{\ln 3}$

$\int_3^\infty \frac{dx}{x(\ln x)^2} = \lim_{b\to\infty}\int_3^b \frac{dx}{x(\ln x)^2} = \lim_{b\to\infty}\left(\frac{-1}{\ln b} + \frac{1}{\ln 3}\right) = 0 + \frac{1}{\ln 3} = 0.91$

$$\int_3^\infty \frac{dx}{x(\ln x)^2} = \frac{1}{\ln 3} = 0.91 \Rightarrow \text{converges}$$

p. 359 8. $\int_1^\infty \frac{1}{e^{st}+2}\,dt$ ~~t~~ Converges and behaves like $1/e^{st}$

If $t \to \infty$, 2 becomes insignificant compared with e^{st}

$$\frac{1}{e^{st}+2} \approx \frac{1}{e^{st}} = e^{-st}$$

$$\int e^{-st}\,dx = \frac{-e^{-st}}{s} = \frac{-1}{se^{st}}$$

$$\int_1^\infty \frac{1}{e^{st}}\,dt = \lim_{b\to\infty} \int_1^b \frac{1}{e^{st}} = \frac{-1}{se^{st}}\Big|_1^b = -\frac{1}{se^{sb}} + \frac{1}{se^{s}} = 0 + \frac{1}{se^s} = \frac{1}{se^s}$$

$\int_1^\infty \frac{1}{e^{st}}\,dt$ converges, so we expect $\int_1^\infty \frac{1}{e^{st}+2}\,dt$ to converge too

$$f(x) = \frac{1}{e^{st}+2}, \quad g(x) = \frac{1}{e^{st}}$$

$$f(x) < g(x)$$

14. $\int_0^1 \frac{1}{x^{19/20}}\,dx = \lim_{a\to 0^+} 20x^{1/20}\Big|_a^1 = 20\cdot 1^{(1/20)} - 20a^{1/20} = 20$

$$\int_0^1 \frac{1}{x^{19/20}}\,dx = 20 \Rightarrow \text{Converges}$$