

Chapter 10

Approximating Functions Using Series

Taylor Polynomials and Taylor Series (Sections 10.1 & 10.2)

For a given infinitely differentiable function $f(x)$, the Taylor polynomial of degree n approximating $f(x)$ near a is

$$P_n(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \cdots + \frac{f^{(n)}(a)}{n!}(x-a)^n$$

For the values of x close to a , $P_n(x) \approx f(x)$.

The Taylor series for $f(x)$ about $x = a$ is the power series

$$P(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \cdots + \frac{f^{(n)}(a)}{n!}(x-a)^n + \cdots$$

The partial sums of the Taylor series are the Taylor polynomials.

For most of the functions $f(x)$ that we are familiar with, their Taylor series do converge to $f(x)$ when x is near a .

Examples of Taylor series: See P. 486, 488.

Announcement on 11/21/06

1. Office hours:
 - Tuesday, Nov. 21, 3 – 4:30 p.m.
 - Wednesday, Nov. 22, 12 - 1:30 p.m.
 - Tuesday, Nov. 28, 1:30 – 3 p.m.
 - Monday, Dec. 4, 12 – 1:30 p.m.
2. No class on Thurs. 11/30, please use time to do exercises in preparation for final exam.
3. For students scoring <70% as of now, you may talk with me to arrange to do some extra exercises to bring your grade up to 70% level before the final exam. This is a completely voluntary exercise.

Lagrange Error Bound for Taylor Polynomial Approximation (Section 10.4, P.498)

Suppose f and all its derivatives are continuous. If $P_n(x)$ is its Taylor polynomial of degree n about $x = a$, then

$$|E_n(x)| = |f(x) - P_n(x)| \leq \frac{M}{(n+1)!} |x - a|^{n+1}$$

where $M \geq |f^{(n+1)}|$ on the interval between a and x .

Announcement on 11/28/06

1. Office hours:
Tuesday, Nov. 28, 1:30 – 3 p.m.
Monday, Dec. 4, 12 – 1:30 p.m.
2. No class on Thurs. 11/30, please use time to do exercises in preparation for final exam.
P. 489 #3, 11; P. 495 #1, 3; P. 501 #1, 3.
3. Final Exam on Tuesday 12/5 at 1:00 - 3:45 p.m. in LART 222.
4. Individually assigned “catch-up” exercises are due before final exam begins.
5. Bioinformatics research opportunities for undergraduates in Spring 2007.
6. Need a volunteer to administer teaching evaluation.