Name: _____

1. Consider the fixed point iteration

$$x_{n+1} = 5 - (4 + c)x_n + cx_n^5. \tag{1}$$

For some values of c, the iterations generated by the above formula converges to $\alpha = 1$ provided x_0 is chosen sufficiently close to α.

(a) Identify the function $g(x)$ which characterizes the above fixed point iteration (1). [That is, the function $g(x)$ satisfying $x_{n+1} = g(x_n)$.]

(b) Find the values of c to ensure the convergence of the iterations generated by the above formula provided x_0 is chosen sufficiently close to α.

(c) For what values of c is this convergence quadratic ?

2. Consider the data $\{(-1, -12), (2, 6), (3, 20)\}$.

   Use Lagrange's formula to find the quadratic polynomial $p_2(x)$ that interpolates the above data. Find the expression in the simplest form.
   You may use the formula: $(x - a)(x - b) = x^2 - (a + b)x + ab$.