Section 9.2

Definitions of Convergent and Divergent Series: For the infinite series $\sum_{n=1}^{\infty} a_n$, the ***n*th partial sum** is given by

$$S_n = a_1 + a_2 + \cdots + a_n.$$

If the sequence of partial sums $\{S_n\}$ converges to S, then the series $\sum_{n=1}^{\infty} a_n$ **converges**. The limit S is called the **sum of the series**. If $\{S_n\}$ diverges, the series **diverges.**

Telescoping Series: A series of the form $(b_1 - b_2) + (b_2 - b_3) + (b_3 - b_4) + (b_4 - b_5) + \cdots$.

Geometric Series: A series of the form

$$\sum_{n=0}^{\infty} ar^n = a + ar + ar^2 + \cdots + ar^n + \cdots, \qquad a \neq 0$$

is a geometric series with ratio r.

Convergence of a Geometric Series: A geometric series with ratio r diverges if $|r| \geq 1$. If $0 < |r| < 1$, then the series converges to the sum

$$\sum_{n=0}^{\infty} ar^n = \frac{a}{1-r}, \qquad 0 < |r| < 1.$$

Limit of the *n*th Term of a Convergent Series: If $\sum_{n=1}^{\infty} a_n$ converges, then $\lim_{n\to\infty} a_n = 0$.

The Divergence Test: If $\lim_{n\to\infty} a_n \neq 0$, then $\sum_{n=1}^{\infty} a_n$ diverges.

1) Write the first three partial sums of the series, and then determine whether the series converges or diverges. If the series converges, find its sum.

$$\sum_{n=1}^{\infty} \frac{1}{3^n}$$

2) Write the series in telescoping form, and then find the sum of the series.

$$\sum_{n=1}^{\infty} \frac{2}{n^2 + 4n + 3}$$

3) Determine whether the geometric series converges or diverges. If it converges, find its sum.

$$\sum_{n=0}^{\infty} \frac{5^{n+1}}{6^n}$$

4) Determine if the following series diverge.

a) $\sum_{n=1}^{\infty} \frac{n}{2n+1}$

b) $\sum_{n=1}^{\infty} \frac{2^n+1}{2^{n+1}}$

5) A ball bearing is dropped from a height of 10 feet and begins bouncing. The height of each bounce is five-sixths the height of the previous bounce. Find the total vertical distance traveled by the ball bearing.

Homework for 9.2: #3, 6, 7, 11, 25, 27, 29, 37, 44, 46, 47, 49