

Math 4329: Numerical Analysis Lecture 01

Natasha S. Sharma, PhD

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Math 4329: Numerical Analysis Lecture 01

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- In the simplest sense, a computational extension of Calculus.
- Most of the Calculus problems will be analyzed from a computational point of view. This means we will study methods and algorithms to approximate the solution to these problems.
- To name a few problems...
 - Evaluation of "complicated" functions at a point-using "simpler" functions;
 - Numerical Differentiation and Integration
 - Finding the x-intercepts of a function.
- Practical Implications:
 - Evaluate the quality of the algorithm in terms of efficiency and accuracy.



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 - **3** Finding the x-intercepts of a function.
- Practical Implications:
 - **1** Evaulate the quality of the algorithm in terms of efficiency and accuracy.
 - 2 Use MATLAB software to solve these problems.



Evaluation of "complicated" functions at a point using "simpler" functions

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Problem: What is the value of e^0 , e^{-1} , $e^{-0.5}$?

Solution:

Use the simpler function Taylor Polynomial to find these values. This has to be based on a evaluation at a known point for example at 0 since we know e⁰ = 1.

- Error in Taylor Polynomial
- Practice Problems for you.



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Natasha S. Sharma, PhD Taylor series is a representation of a function f(x) as an infinite sum of terms that are calculated from the values of the function's derivatives at a single point *a*.

• $f(x) = f(a)/0! + f'(a)(x-a)/1! + f''(a)\frac{(x-a)^2}{2!} + \cdots$

Short Hand Infinite Series Form: $\sum_{n=1}^{\infty} f^n(a) \frac{(x-a)^n}{n!}$

• Example: Taylor Series for $f(x) = e^x$ at a = 0 is

 $f(x) = \frac{x^0}{0!} + \frac{x^1}{1} + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$



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$$f(x) = f(a)/0! + f'(a)(x-a)/1! + f''(a)\frac{(x-a)^2}{2!} + \cdots$$

 "Finite Taylor Series" is a Taylor polynomial obtained by truncating the Taylor Series.

• Example Taylor Polynomial of Degree 1 at a = 0 is

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Taylor Polynomial of degree 1 at 0

• Taylor Series for $f(x) = e^x$ at a = 0 is

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Degree 2 Taylor Polynomial at 0

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Taylor Polynomial of degree 1 at 0

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Natasha S. Sharma, PhD $p_1(x) = \frac{f(a)}{0!} + \frac{f'(a)(x-a)}{1!}$ $p_2(x) = \frac{f(a)}{0!} + \frac{f'(a)(x-a)}{1!} + \frac{f''(a)(x-a)^2}{2!}$ $p_3(x) = \frac{f(a)}{0!} + \frac{f'(a)(x-a)}{1!} + \frac{f''(a)(x-a)^2}{2!} + \frac{f'''(a)(x-a)^3}{3!} \cdots$

Graphical Representation?

 $||| p_1(x)$ is a line. $||| p_2(x)$ is a parabolic function $||| p_3(x)$ is a cubic function.



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Graphical Representation?

 $||| p_1(x) \text{ is a line.}$ $||| p_2(x) \text{ is a parabolic function}$ $||| p_3(x) \text{ is a cubic function.}$



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Graphical Representation?

p₁(x) is a line.
 p₂(x) is a parabolic function.



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$$p_1(x) = \frac{f(a)}{0!} + \frac{f'(a)(x-a)}{1!}$$
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• $p_1(-1) = 1 + \frac{(-1-0)}{1!}$

• $p_2(x) = \frac{f(a)}{0!} + \frac{f'(a)(x-a)}{1!} + \frac{f''(a)(x-a)^2}{2!}$

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$$p_1(x) = \frac{r(a)}{0!} + \frac{r'(a)(x-a)}{1!}$$
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Exercise: Evaluate $e^{-0.5}$



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Natasha S. Sharma, PhD Error = True Value - Approximated Value

 e(x) = f(x) − p_n(x) where,
 e(x) denotes the error at x
 f(x) is the function at x,
 p_n(x) denotes the degree n polynomi
 Problem is we do not know f(x)?
 Error Representation Formula needed

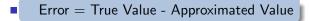
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 c_x an unknown point between a and x



Math 4329: Numerical Analysis Lecture 01

Natasha S. Sharma, PhD



Error Representation Formula needed!

$$f(x) - \rho_n(x) = \frac{(x-a)^{n+1}}{(n+1)!} f^{(n+1)}(c_x).$$

-

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Math 4329: Numerical Analysis Lecture 01

Natasha S. Sharma, PhD Error = True Value - Approximated Value

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