Math 4329: Errors in Solving Linear Systems

Name: $\qquad$

1. Consider the following linear system:

$$
\begin{aligned}
x-\frac{800}{801} y & =10 \\
-x+y & =50
\end{aligned}
$$

Assume that 800/801 $\approx 0.998751560549313$
(a) Verify that the exact solution is $\mathbf{x}^{*}=[48010,48060]^{T}$.
(b) Obtain the solution by a direct inversion of the coefficient matrix and assuming that we are using a computer with four significant digits, compare it with the exact solution $\mathbf{x}^{*}$.
(c) Compare the solution to one obtained on a computer with three digits of significance. Remark on how large the error is for both these cases.
2. Solve the following linear system:

$$
\begin{aligned}
x+y & =2 \\
x+1.0001 y & =2+\alpha .
\end{aligned}
$$

Here the number $\alpha$ assumes the following three values: $0,10^{-3}$ and $10^{-4}$. Can you explain the significant difference in the solutions?

## Solution:

$$
\begin{array}{ll}
\alpha=0 ; \quad x=2 ; \quad y=0 . \\
\alpha=10^{-3} ; \quad x=0.999999999996362 ; & y=1.000000000003638, \\
\alpha=10^{-4} ; \quad x=1.899999999997817 ; \quad y=0.100000000002183 .
\end{array}
$$

