

Homework 1 extension till Thursday 02/13.

$$f(x) = \log(x+2)$$

poly deg n $p_n(x)$ about 0

$$|f(x) - p_n(x)| < 10^{-4} \quad -0.5 \leq x \leq 0.5$$

$$\frac{x^{n+1} f^{(n+1)}(c_x)}{(n+1)!} < 10^{-4}$$

$$f^{(n+1)}(c_x) = ? \rightarrow f^{(n+1)}(x) = \frac{(-1)^n n!}{(x+2)^{n+1}}$$

$n=0$
 $f(x) = \log(x+2)$
 $f'(x) = \frac{1}{x+2}$

$n=0$
 $\frac{(-1)^0 0!}{(x+2)^1}$

$$\frac{x^{n+1}}{(n+1)!}$$

$$\frac{(-1)^n n!}{(c_{x+2})^{n+1}} < 10^{-4}$$

$$\frac{|x|^{n+1}}{(n+1)!} \left| \frac{n!}{(c_{x+2})^{n+1}} \right| < 10^{-4} \quad -0.5 \leq x \leq 0.5$$

$$\frac{0.5^{n+1}}{n+1} \left| \frac{1}{(c_{x+2})^{n+1}} \right| < 10^{-4}$$

$$c_x = ?$$

$$c_x = -0.5? \quad c_x = 0.5? \quad c_x = 0?$$

$$\frac{0.5^{n+1}}{n+1} \left(\frac{2}{3}\right)^{n+1} < 10^{-4}$$

$$2(c) \quad f(x) = e^{\cos x} \quad -0.5 \leq x \leq 0.5$$

$$f'(x) = e^{\cos x} * \frac{d(\cos x)}{dx} \quad \text{chain Rule}$$

$$= e^{\cos x} (-\sin x)$$

$$|f(x) - P_3(x)| = \frac{x^4}{4!} f^{(4)}(c_x)$$

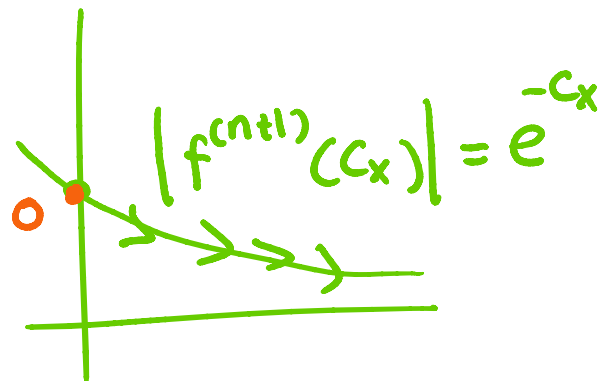
$$\downarrow$$

$$(6\cos c_x + 4\sin c_x) e^{\cos c_x}$$

$$c_x = \pi/3? \quad c_x = \pi/4 \quad c_x = \pi/6?$$

$$f(x) = e^{-x}$$

$$0 \leq x \leq 1$$



$$e^{\cos x}$$

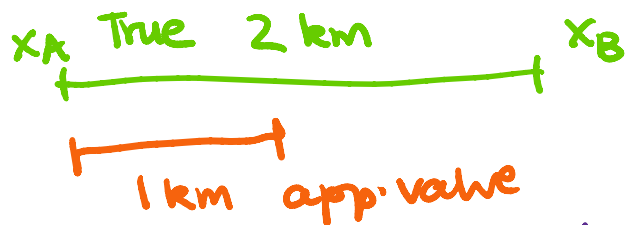
$$e^{\cos x}$$

error = True Value - approximated value

$$|\text{error}| = |\text{True} - \text{approximate value}|$$



error = 1 km 1% error



error = 1 km 50% error.

$$\text{Relative error} = \frac{|\text{True Value} - \text{Approx. Value}|}{|\text{True Value}|}$$

(end of chap 2 slides)

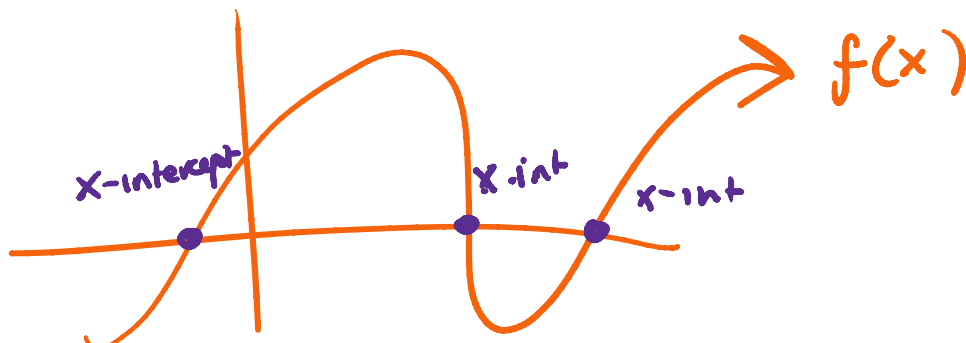
Significant digits

X-intercepts ?

$$f(x) = ax + b$$

$$f(x) = 0$$

$$\Rightarrow x = -b/a$$





$$ax^2 + bx + c = 0$$

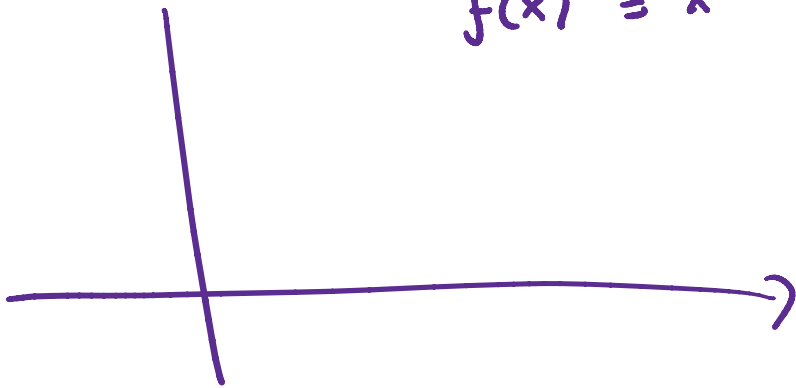
Roots $x_1 = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \rightarrow D$

$$x_2 = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

$\epsilon = 10^{-4} \rightarrow$ accuracy

Iterative methods:

$$f(x) = x^6 - x - 1$$



Pick 2 numbers such that
a and b

$f(a)$ & $f(b)$ are opp. signs.

$f(a) > 0$ & $f(b) < 0$ positive height
negative height

OR $f(a) < 0$ & $f(b) > 0$
negative height positive height

negative height

positive height

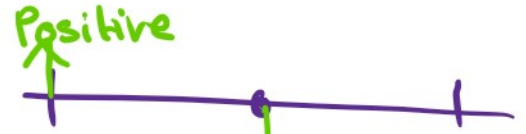
$$f(x) = x^6 - x - 1$$

$$f(0) = -1$$

$$\downarrow \\ b = 0$$

$$f(-1) = (-1)^6 - (-1) - 1 = 1$$

$$\downarrow \\ a = -1$$



$$c = \frac{a+b}{2} = \frac{-1+0}{2} = -0.5$$

$$a = -1$$

$$c = -0.5 \\ \text{negative}$$

$$0 = b \\ \text{negative}$$

$$f(c) = ? \rightarrow f(-0.5) = (-0.5)^6 - (-0.5) - 1$$

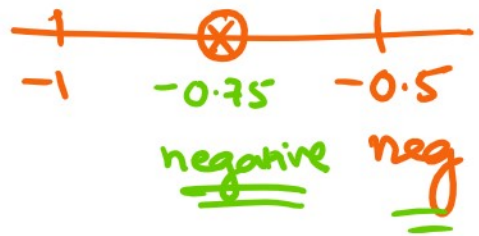
$$0.0156 + 0.5 - 1$$

$$= -0.484375 < 0$$

Replace $b=0$ with -0.5 .

Positive

$$\text{New } a = -1 \quad b = -0.5$$



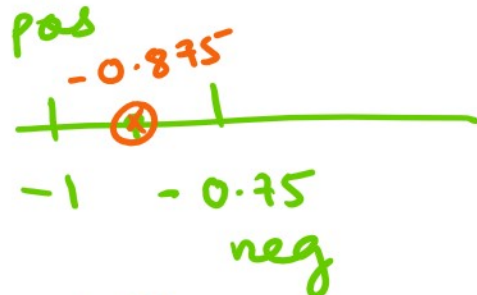
$$c = -0.75$$

$$f(c) = (-0.75)^6 - (-0.75) - 1$$

$$= 0.1779 - 0.25 = -0.072 < 0$$

Replace $b = -0.5$ with -0.75

$$\text{New } a = -1 \quad b = -0.75$$

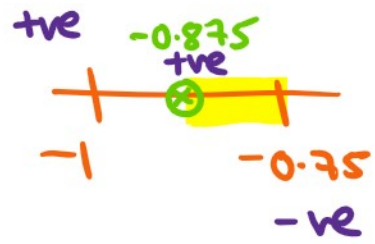


$$c = -0.875 = \frac{1}{2}(a+b)$$

... -0.6 ... -0.7 ...

$$f(-0.875) = (0.875)^6 - (-0.875) + 1$$

$$= 0.323 > 0$$

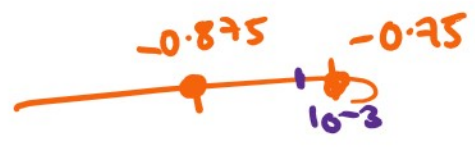


Discard -1 !

& ^{new} $a = -0.875$

new $b = -0.75$

$\epsilon = \text{epsilon} = 10^{-3}$



(Go to slides)

MATLAB COMMAND:

```
>> syms x
>> diff (exp(cos(x)))
ans =
-exp(cos(x))*sin(x)
```