Math 4329 Practice Exam

The problems below are intended to provide students with some practice. Please note that this is not an exhaustive list. The scope of the exam includes all the material covered in the lectures and worksheets as well. No other formulas will be provided.

1. This problem is based on Rootfinding techniques.

(a) Consider the equation $x\ e^x = \cos(x)$. Find the interval $[a, b]$ containing a root α.

(b) Calculate the first three iterations $c_1,\ c_2$ and c_3 of the bisection method using a and b from part (a) and $\epsilon = 10^{-4}$.

More practice problems on page 77, exercises 1 and 2.

2. This question is related to floating-point numbers.

(a) Determine the number x that has the following binary format:

$$(10011\ 1111\ 1001)_2$$

(b) Furthermore, recall the **double** precision representation for any number y is

$$y = \sigma \cdot (1.a_1a_2a_3 \cdots a_{52}) \cdot 2^{E-1023},\ \text{where } E = (c_1c_2c_3 \cdots c_{11})_2.$$

Please express the number x obtained above in its double precision representation.

3. How large should the degree $2n+1$ be chosen in the Taylor expansion

$$\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots + (-1)^n \frac{x^{2n+1}}{(2n+1)!} + (-1)^{n+1} \frac{x^{2n+3}}{(2n+3)!} \sin(c)$$

to have

$$|\sin(x) - p_{2n+1}(x)| \leq 0.01$$

for all $-2\pi \leq x \leq \pi$?

Note: p_{2n+1} denotes the Taylor polynomial of degree $2n+1$ for $f(x)$ about 0 and c denotes a real number between 0 and x.

More practice problems on page 18, exercises 1, 2, 3, 5, 8.

4. This question is concerned with the loss-of-significance errors.
For each of the following functions $f(x)$ evaluated at $x = (0.01)^n$, $n = 1, 2 \cdots 10$, there is a loss of significance that occurs.

$$\text{(a) } f(x) = \frac{\sqrt{4+x} - 2}{x}, \qquad \text{(b) } f(x) = \frac{x - \sin x}{x^3}.$$

Discuss how to avoid the loss of significance in the following calculations by reformulating $f(x)$ as a mathematically equivalent function $g(x)$. Please specify the function $g(x)$.

More practice problems on page 54, exercises 5, 6.

5. Use the Taylor Polynomial of degree 3 for $f(x) = \sqrt{x+1}$ to compute the value of $\sqrt{1.5}$ about 0. **More practice problems on page 9, exercises 5, 6.**
Compute the absolute and relative error assuming that the true value of $\sqrt{1.5} \approx 1.2247$.