Math 2313, Test I

Name _____

1. Find the equation of the plane:

   a. perpendicular to the line $x(t) = -1 + t, y(t) = -2t, z(t) = 3 - 5t$ and through the point $(1, 1, 3)$
   answer: $1(x-1) - 2(y-1) - 5(z-3) = 0$, or $x - 2y - 5z + 16 = 0$

   b. through the points $(0, 0, 0), (-2, 1, 1), (0, 2, 1)$.
   answer: $< -2, 1, 1 > x < 0, 2, 1 > = < -1, 2, -4 >$, so $-x + 2y - 4z = 0$

2. Consider the two planes $2x + 2y - 2z = 3$ and $-x - 2y - z = 5$.

   a. Find parametric equations for the line of intersection of these planes. (Hint: set $z = t$ and solve for $x(t), y(t)$.)
   answer: $x = 3t + 8, y = -2t - \frac{13}{2}, z = t$

   b. Find the angle between the two planes (at the intersection)
   answer: angle between $< 2, 2, -2 >$ and $< -1, -2, -1 >$ is 118.1^o.

3. If $r(t) =< sin(t^2), cos(t^2), t^2 >$, find $r'(t), r''(t)$ and find the magnitude of the velocity, $||r'(t)||$.
   answer: $r'(t) =< 2t\ cos(t^2), -2t\ sin(t^2), 2t >, \|r'(t)\| = \sqrt{8}|t|$
   $r''(t) =< 2cos(t^2) - 4t^2 sin(t^2), -2sin(t^2) - 4t^2 cos(t^2), 2 >$

4. Find the length of the helix of problem 3, from $t = 0$ to $t = 2$.
   answer: $\int_0^2 \|r'(t)\|dt = \int_0^2 \sqrt{8}t\ dt = 2\sqrt{8}$

5. Find parametric equations for the tangent line to the helix of problem 3, at the point $(0, -1, \pi)$ (that is, at time $t = \sqrt{\pi}$).
   answer: $x = -2\sqrt{\pi}t, y = -1, z = \pi + 2\sqrt{\pi}t$, or
   just $x = -t, y = -1, z = \pi + t$