Math 2313, Test I

Name ______________________________

1. Find the equations (parametric or symmetric) of the line:

   a. perpendicular to the plane $2x + 3y - 2z - 8 = 0$ and through the point $(2, 2, 3)$
   answer: $x = 2 + 2t, y = 2 + 3t, z = 3 - 2t$ or $\frac{x-2}{2} = \frac{y-2}{3} = \frac{z-3}{-2}$

   b. through the points $(0, 0, 3), (2, 1, 0)$.
   answer: $(2, 1, 0) - (0, 0, 3) =< 2, 1, -3 >$ so $x = 2t, y = t, z = 3 - 3t$
   (many other forms possible)

2. Consider the two planes $4x - 3y + 2z = 12$ and $x + 5y - z = 25$.

   a. Find a vector parallel to the line of intersection of these planes.
   answer: $(-7, 6, 23)$

   b. Find the angle between the two planes (at the intersection)
   answer: $\theta = 117.68$ (or 62.32) degrees

3. Find the point of intersection (if any) of the plane $2x - 2y + z = 10$ and the line $x = 1 + 4t, y = 2t, z = 3 + 6t$.
answer: $(3, 1, 6)$

4. If $r(t) =< sin(\pi t^2), cos(\pi t^2), t^2 >$, find the velocity $r'(t)$ and the speed, $||r'(t)||$.
answer: $r'(t) =< 2t\pi cos(\pi t^2), -2t\pi sin(\pi t^2), 2t >, \|r'(t)\| = 2|t|\sqrt{\pi^2 + 1}$

5. Find the length of the helix of problem 4, from $t = 0$ to $t = 2$.
answer: $\int_0^2 \|r'(t)\| dt = \int_0^2 2|t|\sqrt{\pi^2 + 1}\; dt = 4\sqrt{\pi^2 + 1}$

6. If the acceleration of an object is $r''(t) =< t, -1 >$, find the position vector $r(t) =< x(t), y(t) >$, if $r(0) =< 0, 1 >$ and $r'(0) =< 1, 0 >$.
answer: $r(t) =< t^3/6 + t, -t^2/2 + 1 >$