

Math 2313, Test II

Name _____

1. If $f(x, y, z) = \ln(x^2y^3z^4)$,
 - a. Find the gradient of f at $(2, 3, 4)$.
answer: $(1, 1, 1)$
 - b. Find the derivative of f at $(2, 3, 4)$ in the direction of the vector $\langle 0, 1, 1 \rangle$.
answer: $\frac{2}{\sqrt{2}}$
 - c. In what direction is the directional derivative smallest (most negative), at the point $(2, 3, 4)$?
answer: $(-1, -1, -1)$
 - d. Find the equation of the tangent plane to the surface $f(x, y, z) = \ln(27648)$ at $(2, 3, 4)$.
answer: $x + y + z = 9$
2. If the temperature in a flat area is given by $T = T(x, y)$ and the coordinates of your car are $x = x(t), y = y(t)$, where t is time, write a formula which gives the rate of change of temperature with respect to time, inside your car.

answer: $\frac{dT}{dt} = \frac{\partial T}{\partial x} \frac{dx}{dt} + \frac{\partial T}{\partial y} \frac{dy}{dt}$

3. If $f(x, y) = \sin(x)\cos(y)$, check each of the following points to see if it is a critical point. If so, classify as a local minimum, local maximum, or saddle point. (Hint: first calculate f_x, \dots, D below)

$$f_x = \cos(x)\cos(y)$$

$$f_y = -\sin(x)\sin(y)$$

$$f_{xx} = -\sin(x)\cos(y)$$

$$f_{yy} = -\sin(x)\cos(y)$$

$$f_{xy} = -\cos(x)\sin(y)$$

$$D = f_{xx} * f_{yy} - (f_{xy})^2 = \sin^2(x)\cos^2(y) - \cos^2(x)\sin^2(y)$$

- a. $x = 0, y = \pi/2$
answer: critical, $D = -1$, saddle point
- b. $x = 0, y = 0$
answer: not critical
- c. $x = -\pi/2, y = 0$
answer: critical, $D = 1, f_{xx} = 1$, minimum
4. Find the point on the plane $3x + 2y + z = 1$ which is closest to the origin $(0, 0, 0)$.

answer: $(\frac{3}{14}, \frac{2}{14}, \frac{1}{14})$