Math 2313, Test II

Name ____________________________

1. If $w(x, y) = x^3 + xy^2 + e^{xy}$, find:

   a. $\frac{\partial w}{\partial y} =$
   answer: $2xy + xe^{xy}$

   b. $w_{xx} + w_{yy} =$
   answer: $8x + (x^2 + y^2)e^{xy}$

2. If $f(x, y, z) = ln(\sqrt{x^3 + y + z^2})$,

   a. Find the gradient of f at $(1, 1, 1)$.
   answer: $< \frac{1}{2}, \frac{1}{6}, \frac{1}{3} >$

   b. Find the derivative of f at $(1, 1, 1)$ in the direction of the vector $< 2, 2, -1 >$.
   answer: $\frac{1}{3}$

   c. In what direction is the directional derivative largest, at the point $(1, 1, 1)$?
   answer: $< \frac{1}{2}, \frac{1}{6}, \frac{1}{3} >$

   d. Find the equation of the tangent plane to the surface $f(x, y, z) = ln(\sqrt{3})$ at $(1, 1, 1)$.
   answer: $3x + y + 2z = 6$

3. If the temperature is $T(x,y) = x^2y^3 + ln(xy)$, what is the rate of change of temperature in a car at $(1,1)$, if the velocity of the car is $(\frac{dx}{dt}, \frac{dy}{dt}) = (2,5)$?
answer: 26

4. Find the point on the plane $z = 12 - 2x - 3y$ closest to $(1,1,1)$. Then prove that this point really minimizes the distance using the second derivative test.
answer: $(\frac{13}{7}, \frac{16}{7}, \frac{10}{7})$
$d_{xx}d_{yy} - d_{xy}^2 = 56 > 0$ and $d_{xx} > 0$ so it's a minimum

5. If $(U_x, U_y, U_z) = (3, 4, -1)$ at the point $(-1, 0, 0)$, which has spherical coordinates $\rho = 1, \phi = \frac{\pi}{2}, \theta = \pi$, find U_θ at this point. For spherical coordinates,

$$\begin{aligned} x &= \rho\ sin(\phi)cos(\theta) \\ y &= \rho\ sin(\phi)sin(\theta) \\ z &= \rho\ cos(\phi) \end{aligned}$$

answer: -4