

Math 2313, Test III

Name \_\_\_\_\_

1. Evaluate  $\int_0^2 \int_0^z \int_0^y x^2 y z^3 dx dy dz$

answer:  $512/135$

2. Find the volume of the cone  $z = 2 - \sqrt{x^2 + y^2}$  above the  $xy$  plane, using a double integral. (Hint: much easier in polar coordinates)

answer:  $8\pi/3$

3. Find the surface area of the cone of problem 2, above the  $xy$  plane (ie, don't include the base of the cone), using a double integral.

answer:  $4\sqrt{2}\pi$

4. Reverse the order of integration:  $\int_0^2 \int_{x^3}^{4x} f(x, y) \, dy \, dx$

answer:  $\int_0^8 \int_{y/4}^{y^{1/3}} f(x, y) \, dx \, dy$

5. a. Find the mass of the cube  $0 \leq x \leq 1, 0 \leq y \leq 1, 0 \leq z \leq 1$ , if the density is  $\rho(x, y, z) = z^2$ .

answer:  $\frac{1}{3}$

b. Find the center of mass of this cube. (Hint: two of the three coordinates can be found by symmetry, without doing any integrals.)

answer:  $(\frac{1}{2}, \frac{1}{2}, \frac{3}{4})$