Math 2313, Test III

Name ______________________________

1. Find the point on the plane $z = 13 - 2x - 3y$ closest to $(1, -1, 0)$. Then prove that this point really minimizes the distance using the second derivative test.
answer: $(3, 2, 1)$
$d_{xx}d_{yy} - d_{xy}^2 = 56 > 0$ and $d_{xx} > 0$ so it's a minimum

2. Reverse the order of integration: $\int_0^9 \int_{\sqrt{x}}^3 f(x,y)dydx$

answer: $\int_0^3 \int_0^{y^2} f(x,y)dxdy$

3. If $\rho(x, y)$ is the density of the lamina below $y = 4 - x^2$ and above the x-axis ($y = 0$), write out the integrals which would need to be evaluated to find the y-coordinate of the center of mass $\bar{y}$.

answer: $\bar{y} = \int_{-2}^2 \int_0^{4-x^2} y\rho(x,y)dy\ dx / \int_{-2}^2 \int_0^{4-x^2} \rho(x,y)dy\ dx$

4. Convert the triple integral $\int_{-3}^{3}\int_{0}^{\sqrt{9-x^2}}\int_{-\sqrt{9-x^2-y^2}}^{\sqrt{9-x^2-y^2}}\sqrt{x^2+y^2+z^2}\,dz\,dy\,dx$ into spherical coordinates and evaluate. (Hint: $x=\rho\ sin(\phi)cos(\theta), y=\rho\ sin(\phi)sin(\theta), z=\rho\ cos(\phi)$ and $dx\ dy\ dz=\rho^2 sin(\phi)d\rho\ d\phi\ d\theta$.)

answer: $\int_0^{\pi}\int_0^{\pi}\int_0^{3}\rho\rho^2 sin(\phi)d\rho\ d\phi\ d\theta=\frac{81\pi}{2}$

5. a. Find the volume of the region below the surface $z=1+\frac{2}{3}x^{3/2}+\frac{2}{3}y^{3/2}$ and above the box $0<x<1, 0<y<1$.

answer: $\frac{23}{15}$

b. Find the surface area of this surface, above the same box.

answer: $\frac{4}{15}(9\sqrt{3}-8\sqrt{2}+1)=1.4066$