Math 2313, Test III

Name ______________________________

1. Consider the half of the sphere $x^2 + y^2 + z^2 = 25$ above the xy plane.

   a. Set up in polar coordinates the double integral to find the volume of this hemisphere. Do not evaluate.

   answer: $\int_0^{2\pi} \int_0^5 \sqrt{25 - r^2} r dr d\theta$

   b. Set up in polar coordinates the double integral to find the surface area of the hemisphere. Do not evaluate.

   answer: $\int_0^{2\pi} \int_0^5 5r/\sqrt{25 - r^2} dr d\theta$

   c. Find the centroid of this hemisphere. You may use the fact that the volume of a half sphere of radius R is $\frac{2}{3}\pi R^3 = \frac{250\pi}{3}$. (Hint: Use cylindrical coordinates.)

   answer: 15/8

2. Evaluate $\int_0^2 \int_0^z \int_0^y x^3 y^3 z^3 \; dx \; dy \; dz$

answer: 32/3

3. Reverse the order of integration: $\int_0^8 \int_{y/4}^{y^{1/3}} f(x,y) \; dx \; dy$

answer: $\int_0^2 \int_{x^3}^{4x} f(x,y) \; dy \; dx$

4. Find x, y, z such that $f(x,y,z) = 3x + 4y + 6z$ is minimized, with the restriction that $xyz = 24$, and all three variables are positive. For extra credit: find the discriminant $f_{xx}f_{yy} - f_{xy}^2$ at your critical point to see if you have found a saddle point or not.

answer: $(4,3,2), f_{xx}f_{yy} - f_{xy}^2 = 3$