Math 2313, Test III

Name ______________________________

1. Find the point on the plane $z = 1 + 2x + y$ closest to $(1, 1, 1)$. Then prove that this point really minimizes the distance using the second derivative test.
   answer: $(0, \frac{1}{2}, \frac{3}{2})$
   $d_{xx}d_{yy} - d_{xy}^2 = 24 > 0$ and $d_{xx} > 0$ so it's a minimum

2. For the function $p(x, y) = ae^{-x^2-y^2}$

   a. For what value of a is this a joint probability distribution? (Hint: use polar coordinates)

   answer: $a = \frac{1}{\pi}$

   b. Write an integral (don't evaluate it) which expresses the probability that $x > 3$.

   answer: $\int_3^\infty \int_{-\infty}^\infty p(x, y) dy dx$

3. Consider the half of the sphere $x^2 + y^2 + z^2 = 36$ above the xy plane.

   a. If the density is $\rho = \sqrt{x^2 + y^2 + z^2}$, set up **in spherical coordinates** and evaluate a triple integral to find the mass of this hemisphere. (Hint: $dx\ dy\ dz = \rho^2 sin(\phi)\ d\rho\ d\phi\ d\theta$)

      answer: $M = 648\pi$

   b. Set up, in polar coordinates, a double integral to find the surface area of this hemisphere. Do not evaluate the integral.

      answer: $\int_0^{2\pi} \int_0^6 6r/\sqrt{36 - r^2} dr d\theta$

4. Reverse the order of integration in $\int_0^1 \int_y^1 cos(x^2) dx dy$, and evaluate the new integral.

   answer: $sin(1)/2$