Math 2313, Final

Name ______________________________

1. Consider the surface $x^3 ln(y) + ze^{xz} = 0$:

   a. Write the equation for the tangent plane at the point $(1, 1, 0)$.

   answer: $y + z = 1$

   b. Write the parametric equations for the normal line at $(1, 1, 0)$.

   answer: $(x, y, z) = (1, 1 + t, t)$

2. Consider a particle whose position is given by the vector $r(t) =< cos(3t), sin(3t), 4t >$. Find the velocity $r'(t)$, speed $\|r'(t)\|$, and acceleration $r''(t)$. Show that the acceleration is always perpendicular to the velocity.

   answer: $r' =< -3sin(3t), 3cos(3t), 4 >, \|r'\| = 5$
   $r'' =< -9cos(3t), -9sin(3t), 0 >$
   $r' \bullet r'' = 0$

3. Express $(\frac{\partial u}{\partial r})^2 + \frac{1}{r^2}(\frac{\partial u}{\partial \theta})^2$ in terms of $\frac{\partial u}{\partial x}$ and $\frac{\partial u}{\partial y}$, using the chain rule, and simplify as much as possible. (Hint: $x = r\ cos(\theta), y = r\ sin(\theta)$)

answer: $(\frac{\partial u}{\partial x})^2 + (\frac{\partial u}{\partial y})^2$

4. Find the dimensions of an open box (no top) with a volume of $32cm^3$ which uses the least amount of cardboard.

answer: $x = 4cm, y = 4cm, z = 2cm$

5. Evaluate $\int_0^1 \int_y^1 \sqrt{1 - x^2} dx\ dy$. (Hint: you may need to reverse the order of integration.)

answer: $\frac{1}{3}$

6. Write an integral, using x, y coordinates, which gives the volume of a sphere of radius a, $z = \sqrt{a^2 - x^2 - y^2}$, in the first octant $(x \geq 0, y \geq 0, z \geq 0)$. Then convert the integral to polar coordinates and evaluate it.

answer: $\frac{\pi a^3}{6}$

7. A sphere of radius 4 has density given by $\rho(x, y, z) = e^z$. Set up the integrals required to compute the z-coordinate of the center of mass, $\bar{z}$ ($\bar{x}$ and $\bar{y}$ are zero by symmetry.) Don't evaluate the integrals.

answer: $M = \int_0^{2\pi} \int_0^4 \int_{-\sqrt{16-r^2}}^{\sqrt{16-r^2}} e^z r \; dz \; dr \; d\theta$
$\bar{z} = \frac{1}{M} \int_0^{2\pi} \int_0^4 \int_{-\sqrt{16-r^2}}^{\sqrt{16-r^2}} z e^z r \; dz \; dr \; d\theta$