Math 2313, Final

Name ____________________________

1. Find a vector perpendicular to both of the lines $x = 1+t, y = 2+t, z = 3+t$ and $x = 1, y = 2+t, z = -2+t$.

   answer: $< 0, -1, 1 >$

2. Find the equation of the normal line to the surface $x^2 + y^2 + z^2 = 9$ at $(-1, 2, 2)$.

   answer: $x = -1 - 2t, y = 2 + 4t, z = 2 + 4t$

3. **Use the chain rule** to find $\frac{\partial U}{\partial p}$ if
   $U = x^3 + ln(xy) + e^{3yz}$
   $x = pq$
   $y = p/q$
   $z = p^2 + q^2$

   answer: $(3x^2 + \frac{1}{x})q + (\frac{1}{y} + 3ze^{3yz})(\frac{1}{q}) + (3ye^{3yz})(2p)$

4. Evaluate $\int_{-1}^{1} \int_{0}^{\sqrt{1-x^2}} e^{x^2+y^2} dydx$

   answer: $\frac{\pi}{2}(e - 1)$

5. Find the directional derivative of $f(x, y, z) = x^3 + ln(xy) + e^{3yz}$ at the point $(1, 1, 0)$ in the direction of the vector $< -1, -1, 2 >$.

answer: $\frac{1}{\sqrt{6}}$

6. Find the point on the surface $z = \sqrt{1 - 2x - 4y}$ which is closest to the origin $(-3, -5, 0)$.

answer: $(-2, -3, \sqrt{17})$

7. Write an integral which, if evaluated (but don't evaluate), would give the mass of the solid in the first octant inside the ellipsoid $3x^2+y^2+z^2 = 27$, if the density is given by $\rho(x, y, z)$.

answer: $\int_0^3 \int_0^{\sqrt{27-3x^2}} \int_0^{\sqrt{27-3x^2-y^2}} \rho(x, y, z) dzdydx$