Math 2313, Final

Name ______________________________

1. Find a vector parallel to the plane $x + 2y - z = 0$ and perpendicular to the line $x = -2t, y = 1 + 3t, z = 2$ (Hint: a vector parallel to a plane is perpendicular to its normal vector.)

   answer: $(3, 2, 7)$

2. Find $\frac{\partial U}{\partial p}$ **using the chain rule** if
   $U = ln(xy) + e^{3xy^2}$
   $x = pq$
   $y = 3p + 2q$

   answer: $(\frac{1}{x} + 3y^2e^{3xy^2})(q) + (\frac{1}{y} + 6xye^{3xy^2})(3)$

3. Evaluate $\int_{-3}^{3}\int_{0}^{\sqrt{9-x^2}} cos(x^2 + y^2)dydx$. (Hint: convert to polar coordinates.)

   answer: $\frac{\pi\ sin(9)}{2}$

4. Find the directional derivative of $f(x, y) = ln(xy) + e^{3xy^2}$ at the point $(1, 1)$ in the direction of the vector $(1, -1)$.

answer: $\frac{-3e^3}{\sqrt{2}}$

5. Find the point on the surface $z = \sqrt{x^2 + 2y^2}$ closest to the point $(1, 1, 0)$.

answer: $(\frac{1}{2}, \frac{1}{3}, \sqrt{\frac{17}{36}})$

6. Write an integral which, if evaluated (but don't evaluate), would give the mass of the part of the surface $z = 4 - x^2 - y^2$ above the xy plane, if the density is given by $\rho(x, y, z) = x^2 + y^2 + z^2$. Write the integral in **both** rectangular and cylindrical coordinates.

answer: $\int_{-2}^{2} \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \int_{0}^{4-x^2-y^2} (x^2 + y^2 + z^2) dz dy dx$
$\int_{0}^{2\pi} \int_{0}^{2} \int_{0}^{4-r^2} (r^2 + z^2) r dz dr d\theta$

7. Find the length of the helix $x(t) = cos(t^2), y(t) = t^2, z(t) = sin(t^2)$ from $t = 0$ to $t = 5$.

answer: $25\sqrt{2}$