Math 2313, Final

Name ______________________________

1. Find the equation of the plane that contains the lines $x = -1 + 2t, y = 1 + 3t, z = 2 + 4t$ and $x = -1 + 3s, y = 1 - s, z = 2 + 5s$. (Hint: the lines intersect when $t = 0, s = 0$.)

   answer: $19x + 2y - 11z + 39 = 0$

2. Find the length of the curve with parametric equations $x(t) = cos(t^3), y(t) = sin(t^3), z(t) = t^3$, from $t = 0$ to $t = 3$.

   answer: $27\sqrt{2}$

3. If x changes by $dx = 0.01, z$ changes by $dz = -0.02$ and y does not change, approximately how much does $f(x, y, z)$ change, if the gradient of f at (x, y, z) is $\nabla f =< 4, -3, -2 >$?

   answer: $df = 0.08$

4. Reverse the order of integration in $\int_0^2 \int_{\frac{y}{2}}^{y} (x^2 y) dx dy + \int_2^4 \int_{\frac{y}{2}}^{2} (x^2 y) dx dy$, resulting in a single integral over a triangle. Evaluate the new integral.

answer: $\int_0^2 \int_x^{2x} (x^2 y) dy dx = 9.6$

5. Find all critical points of $f(x, y, z) = 2x^3 - 15x^2 + 36x + z^2 x + 2y^3 + 21y^2 + 72y + 19$.

answer: $(2, -3, 0), (2, -4, 0), (3, -3, 0), (3, -4, 0)$

6. Find the mass of the cylinder $x^2 + y^2 \leq 9, 0 \leq z \leq 4$ whose density is given by $\rho(x, y, z) = x^2 + y^2 + z^2$. (Hint: use cylindrical coordinates.)

answer: 354π

7. Find the directional derivative of $f(x, y) = e^{y^2 x^3}$ at the point $(1, 2)$ in the direction of the vector $< 12, -5 >$.

answer: $\frac{124}{13} e^4$