Math 2313, Final

Name ______________________________

1. Find a vector parallel to the intersection of the planes $x + 2y - z = 0$ and $y + z = 3$.

   answer: $(3, -1, 1)$

2. a. Find the equation of the tangent plane to the surface $2x^2+y^2+z^2 = 15$ at $(1, 3, 2)$.

      answer: $4x + 6y + 4z = 30$

   b. Find the equations of the normal line to this surface at this point.

      answer: $x = 1 + 4t, y = 3 + 6t, z = 2 + 4t$

3. Find $\frac{\partial U}{\partial \theta}$ at $\rho = 1, \phi = \pi/2, \theta = \pi$, if $x = \rho\, sin(\phi)cos(\theta), y = \rho\, sin(\phi)sin(\theta), z = \rho\, cos(\phi)$ and $(U_x, U_y, U_z) = (2, 3, -1)$ at this point.

   answer: -3

4. Evaluate $\int_0^2 \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} e^{x^2+y^2} dydx$

   answer: $\frac{\pi}{2}(e^4 - 1)$

5. Find the directional derivative of $f(x, y, z) = x^3 + ln(xy) + e^{3yz}$ at the point $(1, 1, 0)$ in the direction of the vector $(1, 1, -1)$.

answer: $\frac{2}{\sqrt{3}}$

6. Find the point on the surface $z = \sqrt{x^2 + 2y^2}$ closest to the point $(1, 1, 0)$.

answer: $(\frac{1}{2}, \frac{1}{3}, \sqrt{\frac{17}{36}})$

7. Write an integral which, if evaluated (but don't evaluate), would give the mass of the tetrahedron in the first octant under the plane $6x + 3y + z = 12$, if the density is given by $\rho(x, y, z) = x^2 + z^2$.

answer: $\int_0^2 \int_0^{4-2x} \int_0^{12-6x-3y} (x^2 + z^2) dz dy dx$

8. Find the total distance travelled from $t = 0$ to $t = 3$ by a particle whose position is given by the vector $r(t) =< cos(3t), sin(3t), 4t >$.

answer:15