

Math 2326, Test I

Name _____

For problems 1-3, solve the differential equations. If you cannot solve for the dependent variable, leave the solution defined implicitly. If no initial condition is given, write the general solution.

1. $\frac{dy}{dx} = 3x^2(1 + y^2)$, with $y(0) = 1$

answer: $y = \tan(x^3 + \pi/4)$

2. $\frac{dy}{dt} = \frac{t}{3y^2 + 2y - 1}$, with $y(0) = 1$

answer: $y^3 + y^2 - y = \frac{1}{2}t^2 + 1$

3. $\frac{dy}{dx} + 2y = 6x^2$.

answer: $y = Ce^{-2x} + 3x^2 - 3x + \frac{3}{2}$

4. For the differential equation $y' = e^{3xy}$, $y(1) = 2$, take one step using Euler's method with $h = 0.01$, to approximate $y(1.01)$.

answer: $y(1.01) \approx 6.034$

5. Refer to the direction field plot on the last page to answer the following questions:

a. (Multiple choice) The differential equation whose field is plotted could be:

1. $P' = (P - 1)(P - 4)$

2. $P' = (P + 1)(P - 4)$

3. $P' = (P + 1)(4 - P)$

4. $P' = (t - 1)(t - 4)$

answer: 3

b. There is a stable equilibrium (sink) at what value of P ?

answer: $P = 4$

c. The solution to this differential equation with $P(0) = -1$ converges to what value of P , as $t \rightarrow \infty$?

answer: -1

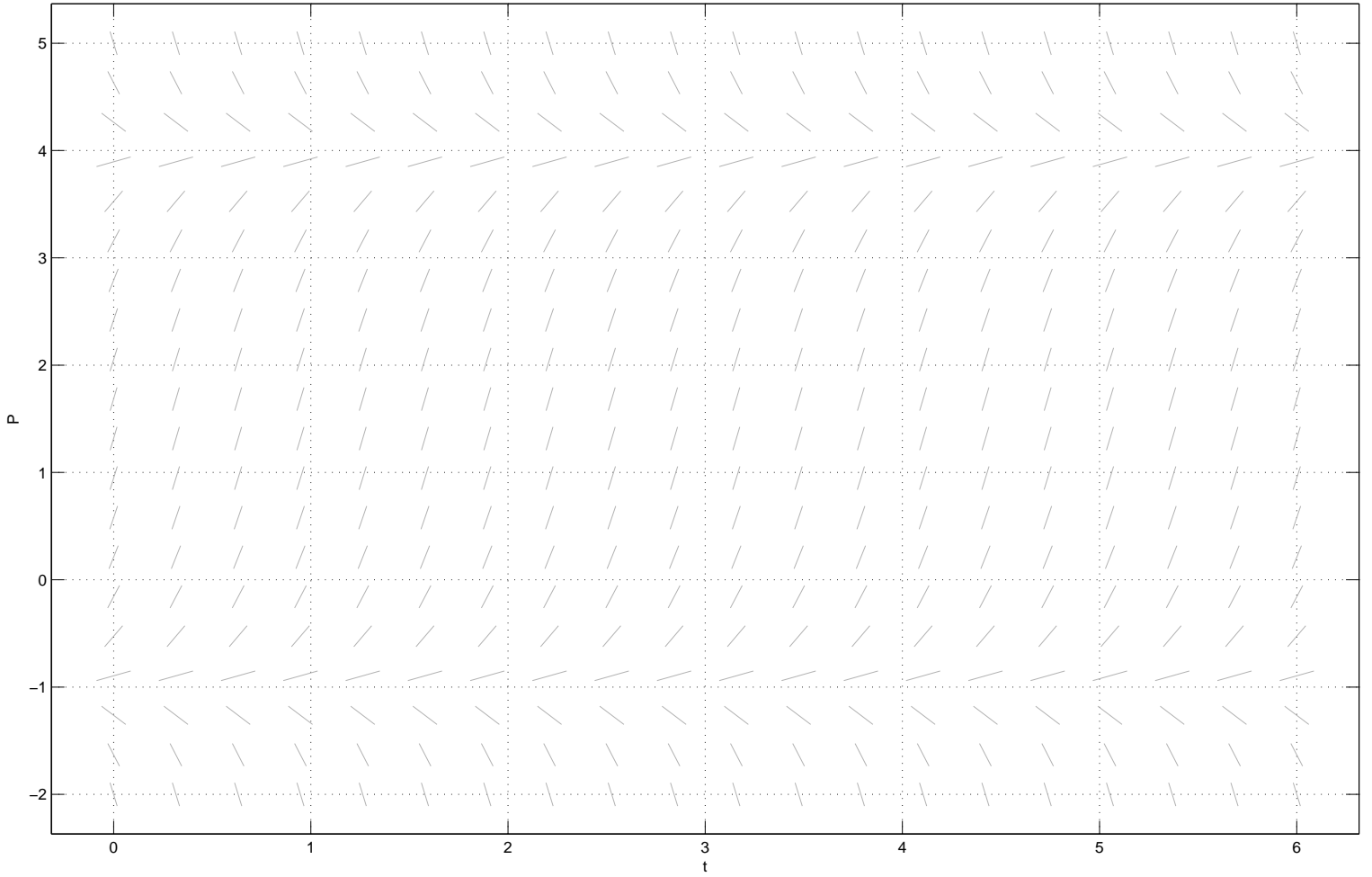
d. The solution with $P(0) = -0.99$ converges to what value of P ?

answer: 4

6. A 50 gallon tank is full to the brim with pure water, and 5 gallons/minute of a brine solution with 0.3 kg/gallon salt flows into it. Since the tank is full, 5 gallons/minute of well-mixed solution flows onto the ground. Find $S(t)$, the number of kg of salt in the tank, as a function of time. What happens to $S(t)$ as $t \rightarrow \infty$?

answer: $S(t) = 15 - 15e^{-0.1t}$; $S(\infty) = 15$

$$P' = (1 + P)(4 - P)$$



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