Math 2326, Test I

Name ______________________________

For problems 1-3, solve the differential equations. If you cannot solve for the dependent variable, leave the solution defined implicitly. If no initial condition is given, write the general solution.

1. $\frac{dy}{dt} = 3sin(9t)(1 + y)$, with $y(0) = 0$

   answer: $y(t) = e^{(1-cos(9t))/3} - 1$

2. $\frac{dy}{dt} = \frac{t}{5y^4+6y^2-4y}$, with $y(1) = 1$

   answer: $y^5 + 2y^3 - 2y^2 = \frac{1}{2}t^2 + \frac{1}{2}$

3. $\frac{dy}{dt} + 2y = 6cos(2t)$

   answer: $y(t) = Ce^{-2t} + \frac{3}{2}sin(2t) + \frac{3}{2}cos(2t)$

4. Consider the linear differential equation:
   $\frac{dy}{dt} + 3y = t^2 + e^{-3t}$

   a. Find the homogeneous solution.
      answer: $y_h(t) = Ce^{-3t}$

   b. A particular solution of the nonhomogeneous equation will have what form? You need not actually find the undetermined constants, just leave them as letters.

answer: $y_p(t) = at^2 + bt + d + ete^{-3t}$

c. Write the general solution of the original (nonhomogeneous) equation. Again, you need not specify values for the undetermined constants.
answer: $y(t) = Ce^{-3t} + at^2 + bt + d + ete^{-3t}$

d. If you have time, **for extra credit**, actually find the constants in part [b].
answer: $a = \frac{1}{3}, b = \frac{-2}{9}, d = \frac{2}{27}, e = 1$

5. Find all equilibrium points of $\frac{dy}{dt} = 1 + cos(\pi y)$ and classify each as a source, sink or node. Then tell what happens to $y(t)$ as $t \to \infty$, if $y(0) = 6$.
answer: $y =$ all odd integers, all are nodes. $y \to 7$ as $t \to \infty$.

6. A cup of coffee is initially at 130^o F and is left in a room with a temperature of 70^o F. Suppose that at time $t = 0$ it is cooling at a rate of 10^o F per minute. Assume that Newton's law of cooling applies, that is, the rate of cooling is proportional to the difference between the current temperature $T(t)$ and the room temperature, 70^o F. Write a differential equation with initial condition for the temperature $T(t)$ and solve it.
answer: $T'(t) = -k(T - 70), T(0) = 130, T(t) = 70 + 60e^{-t/6}$