Math 2326, Test I

Name ______________________________

For problems 1-4, solve the differential equations. If you cannot solve for the dependent variable, leave the solution defined implicitly. If no initial condition is given, write the general solution.

1. $\frac{dy}{dx} - 2y = 4x.$

   answer: $y = Ce^{2x} - 2x - 1$

2. $\frac{dy}{dt} = \frac{t}{3y^2-1}$, with $y(0) = 1$

   answer: $y^3 - y = \frac{1}{2}t^2$

3. $\frac{dy}{dx} = \frac{3x\sqrt{1+y^2}}{y}$, with $y(0) = -1$

   answer: $y = -\sqrt{(\frac{3}{2}x^2 + \sqrt{2})^2 - 1}$

4. $\frac{dy}{dt} + 5y = 3e^{-5t}.$

   answer: $y = Ce^{-5t} + 3te^{-5t}$

5. For the differential equation $y' = x + 2y, y(0) = 1$, take three steps using Euler's method with $h = 0.25$, to approximate $y(0.75)$. You may use the following table, if you want:

x	y	f(x,y) = x+2y	y + h*f(x,y)
0.00	1.000		
0.25	1.5		
0.50	2.3125		
0.75	3.59375	(skip)	(skip)

6. Consider the differential equation $\frac{dP}{dt} = (\frac{P}{200} - 1)(1 - \frac{P}{50})P$

   a. Is this equation autonomous? (answer: yes)

   b. Is this equation linear? (answer: no)

   c. Find all equilibrium points, and classify each as a source, sink or node.
   answer: $P = 200$ is sink, $P = 50$ is source, $P = 0$ is sink

7. A 200 gallon tank is full to the brim with pure water, and 10 gallons/minute of a brine solution with 0.5 kg/gallon salt flows into it. Since the tank is full, 10 gallons/minute of well-mixed solution overflows onto the ground. If $S(t)$ is the number of kg of salt in the tank, as a function of time, find a differential equation with boundary conditions for $S(t)$ (do not solve it).

answer: $S'(t) = 0.5(10) - 10\frac{S}{200}$, with $S(0) = 0$