Math 2326, Test I

Name ______________________________

For problems 1-3, solve the differential equations. If you cannot solve for the dependent variable, leave the solution defined implicitly. If no initial condition is given, write the general solution.

1. $\frac{dy}{dx} = \frac{3x^2}{1+3y^2}$, with $y(2) = 1$

   answer: $y + y^3 = x^3 - 6$

2. $\frac{dy}{dt} = e^{2t-3y}$, with $y(0) = 0$

   answer: $y = \frac{1}{3} ln(\frac{3}{2}e^{2t} - \frac{1}{2})$

3. $\frac{dy}{dt} - 2y = 4e^{2t}$.

   answer: $y = Ce^{2t} + 4te^{2t}$

4. For the differential equation $y' = t^2 - y^2, y(1) = 2$, take one step using Euler's method with $h = 0.01$, to approximate $y(1.01)$.

   answer: $y(1.01) \approx 1.97$

5. Refer to the direction field plot on the last page to answer the following questions:

a. (Multiple choice) The differential equation whose field is plotted could be:

   1. $y' = 1 - cos(\pi y)$
   2. $y' = y(4 - y^2)$
   3. $y' = y(2 + y)$
   4. $y' = y(2 + y)^2$
   5. $y' = -1 + cos(\pi y)$
   6. $y' = sin(\pi y)$

   answer: 1

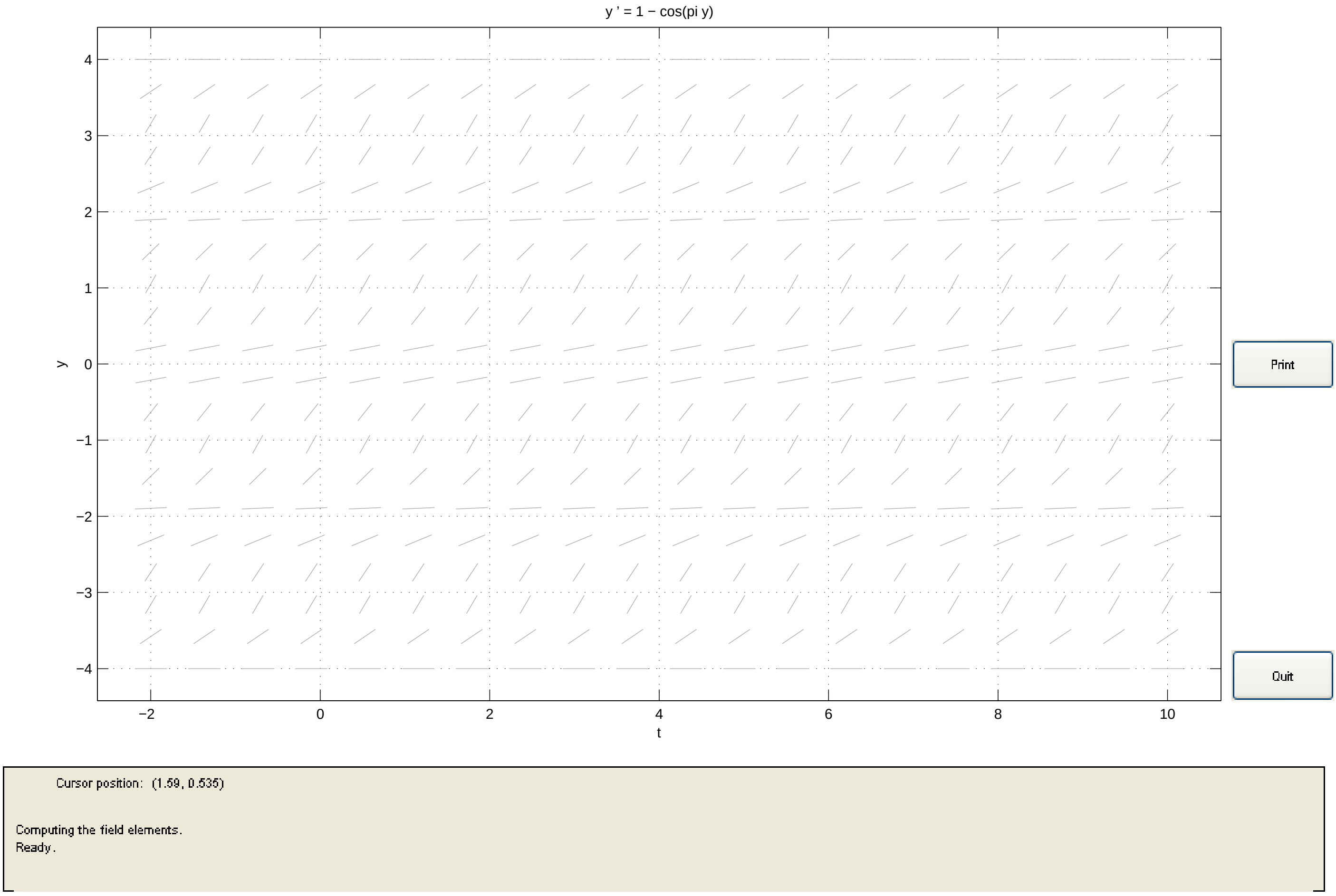
b. Classify the equilibrium point at $y = -2$ as a source, sink or node.
   answer: node

c. The solution to this differential equation with $y(0) = -2$ converges to what value of y, as $t \to \infty$?
   answer: -2

d. The solution with $y(0) = -1.9$ converges to what value of y?
   answer: 0

6. A wine chilled at $5^o C$ is suddenly moved to a room where the temperature is $25^o C$. If at $t = 0$ the wine is warming up at the rate of $5^o C/hour$, when will the wine reach $10^o C$? Assume Newton's law of cooling, that is, $\frac{dT}{dt} = -k(T - T_0)$, where T is temperature, t is time, k is a constant, and T_0 is the room temperature.

answer: $t = 1.15$ hours


y ' = 1 − cos(pi y)
4
3
2
1
0
−1
−2
−3
−4
y
−2
0
2
4
6
8
10
t
Print
Quit
Cursor position: (1.59, 0.535)
Computing the field elements.
Ready.