

Math 2326, Test I

Name _____

1. Solve $\frac{dy}{dt} = y^3$, with $y(0) = 1$

answer: $y(t) = \sqrt{\frac{1}{1-2t}}$

For problems 2-3, find the general solution.

2. $\frac{dy}{dt} + 2y = 3e^{4t} + 6t + 11$

answer: $y(t) = Ae^{-2t} + \frac{1}{2}e^{4t} + 3t + 4$

3. $\frac{dP}{dt} + 3P = \cos(t)$

answer: $P(t) = Ae^{-3t} + 0.3\cos(t) + 0.1\sin(t)$

4. For the differential equation $y' = xy$, $y(1) = 10$, take one step using Euler's method with $h = 0.001$, to approximate $y(1.001)$.

answer: 10.01

5. The solution of $y' = -1 + \cos(y)$, $y(0) = -0.1$ converges to what, as $t \rightarrow \infty$?

answer: $y(\infty) = -2\pi$

6. A 10-gallon bucket is full of pure water. Suppose we dump 2 pounds of salt per minute into the bucket, and at the time we start dumping salt, the bucket develops a leak, through which 0.5 gallons/minute of well-mixed salt water escapes. Write a differential equation, with initial condition, for the number of pounds of salt $S(t)$ in the bucket (do not solve). (Hint: note that the amount of water in the bucket is not constant, first write a formula for the amount of water in the bucket as a function of time.)

answer: $\frac{dS}{dt} = 2 - 0.5\frac{S}{10-0.5t}$

7. Refer to the direction field plot on the last page to answer the following questions:

a. (Multiple choice) The differential equation whose field is plotted could be:

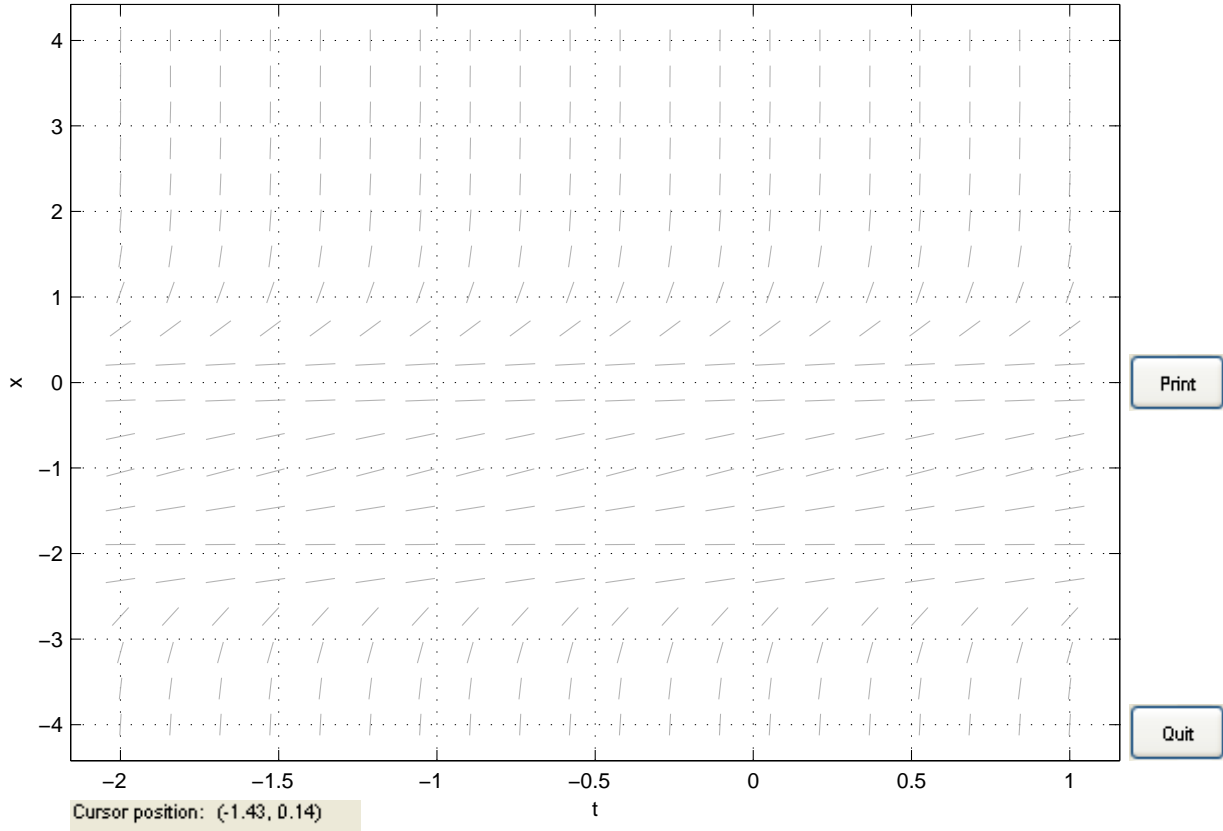
1. $X' = X(X - 2)$
2. $X' = X(X + 2)$
3. $X' = X(X + 2)^2$
4. $X' = X^2(X + 2)$
5. $X' = X^2(X + 2)^2$

answer: 5

b. Classify each equilibrium point as a sink, source or node.

answer: 0, -2 are both nodes

$$x' = x^2(x+2)^2$$



Computing the field elements.
Ready.
Computing the field elements.
Ready.