

## Math 2326, Test I

Name \_\_\_\_\_

For problems 1-3, solve the differential equations. If you cannot solve for the dependent variable, leave the solution defined implicitly. If no initial condition is given, write the general solution.

1.  $\frac{dy}{dx} = 3x^2(1 + y^2)/y$ , with  $y(0) = 1$

answer:  $y = \sqrt{2e^{2x^3} - 1}$

2.  $\frac{dy}{dt} = 2y + \sin(2t)$

answer:  $y = Ce^{2t} - \frac{1}{4}\sin(2t) - \frac{1}{4}\cos(2t)$

3.  $\frac{dy}{dx} + y = 6e^{-x}$

answer:  $y = Ce^{-x} + 6xe^{-x}$

4. Find two **different** solutions of  $y' = 3y^{2/3}$  with  $y(0) = 0$ . Explain why this does not violate the uniqueness theorem.

answer:  $y = 0$  and  $y = t^3$ .  $\frac{\partial f}{\partial y}$  is not continuous at  $y = 0$

5. Consider the differential equation  $P' = (P + 2)^2(P - 5)$ .

a. Is it autonomous? Is it linear? Is it separable?

answer: autonomous and separable, not linear

b. Find all equilibrium points and classify each as a source (unstable), sink (stable) or node (stable on one side, unstable on the other).

answer:  $P = 5$  is source,  $P = -2$  is node

6. For the differential equation  $y' = x + y$ ,  $y(0) = 1$ , take three steps using Euler's method with  $h = 0.25$ , to approximate  $y(0.75)$ . You may use the following table, if you want:

x	y	f(x,y) = x+y	y + h*f(x,y)
0.00	1.000		
0.25	1.250		
0.50	1.625		
0.75	2.15625	(skip)	(skip)

7. The quantity  $I(t)$  of iodine isotope I-131 present satisfies the differential equation  $I'(t) = -0.08664I$ , where  $t$  is in days. What is the half-life of I-131?

answer: 8 days