Math 2326, Test I

Name ______________________________

For problems 1-3, solve the differential equations. If you cannot solve for the dependent variable, leave the solution defined implicitly. If no initial condition is given, write the general solution.

1. $\frac{dy}{dt} = 8cos(4t)(1+y)$, with $y(0) = 0$

   answer: $y(t) = e^{2sin(4t)} - 1$

2. $\frac{dy}{dt} = \frac{t}{5y^4+9y^2-6y}$, with $y(1) = 1$

   answer: $y^5 + 3y^3 - 3y^2 = \frac{1}{2}t^2 + \frac{1}{2}$

3. $\frac{dy}{dt} + 2y = 12cos(2t)$

   answer: $y(t) = Ce^{-2t} + 3sin(2t) + 3cos(2t)$

4. Consider the linear differential equation:
   $\frac{dy}{dt} + 4y = t^3 + e^{-4t}$

a. Find the homogeneous solution.
answer: $y_h(t) = Ce^{-4t}$

b. A particular solution of the nonhomogeneous equation will have what form? You need not actually find the undetermined constants, just leave them as letters.
answer: $y_p(t) = at^3 + bt^2 + dt + e + fte^{-4t}$

c. Write the general solution of the original (nonhomogeneous) equation. Again, you need not specify values for the undetermined constants.
answer: $y(t) = Ce^{-4t} + at^3 + bt^2 + dt + e + fte^{-4t}$

5. Find all equilibrium points of $\frac{dy}{dt} = -1 + cos(\pi y)$ and classify each as a source, sink or node. Then tell what happens to $y(t)$ as $t \to \infty$, if $y(0) = 3$.

answer: $y =$ all even integers, all are nodes. $y \to 2$ as $t \to \infty$.

6. A cup of coffee is initially at 130^o F and is left in a room with a temperature of 70^o F. Suppose that at time $t = 0$ it is cooling at a rate of 10^o F per minute. Assume that Newton's law of cooling applies, that is, $\frac{dT}{dt} = -k(T - T_0)$ where $T_0 = 70^o F$ is the room temperature, and k is a constant. Find the temperature of the cup $T(t)$ as a function of time.

answer: $T(t) = 70 + 60e^{-t/6}$