

Math 2326, Test I

Name _____

For problems 1-3, solve the differential equations. If you cannot solve for the dependent variable, leave the solution defined implicitly. If no initial condition is given, write the general solution.

1. $\frac{dy}{dx} = 3x^2(1 + y^2)$, with $y(0) = -1$

answer: $y = \tan(x^3 - \pi/4)$

2. $\frac{dy}{dt} = \frac{3t^2}{4y^3+1}$, with $y(1) = 1$

answer: $y^4 + y = t^3 + 1$

3. $\frac{dy}{dt} + 2y = 12\cos(2t)$.

answer: $y = Ce^{-2t} + 3\cos(2t) + 3\sin(2t)$

4. For the differential equation $y' = 3x + xy^3$, $y(1) = 2$, take one step using Euler's method with $h = 0.01$, to approximate $y(1.01)$.

answer: $y(1.01) \approx 2.11$

5. Refer to the direction field plot on the last page to answer the following questions:

a. (Multiple choice) The differential equation whose field is plotted could be:

1. $P' = (P - 1)(P - 4)$

2. $P' = (P + 1)(P - 4)$

3. $P' = (P + 1)^2(4 - P)$

4. $P' = (P + 1)(4 - P)$

5. $P' = (t + 1)(P - 4)$

6. $P' = (P + 1)(P - 4)^2$

answer: 4

b. There is a stable equilibrium (sink) at what value of P ?

answer: $P = 4$

c. The solution to this differential equation with $P(1) = -1$ converges to what value of P , as $t \rightarrow \infty$?

answer: -1

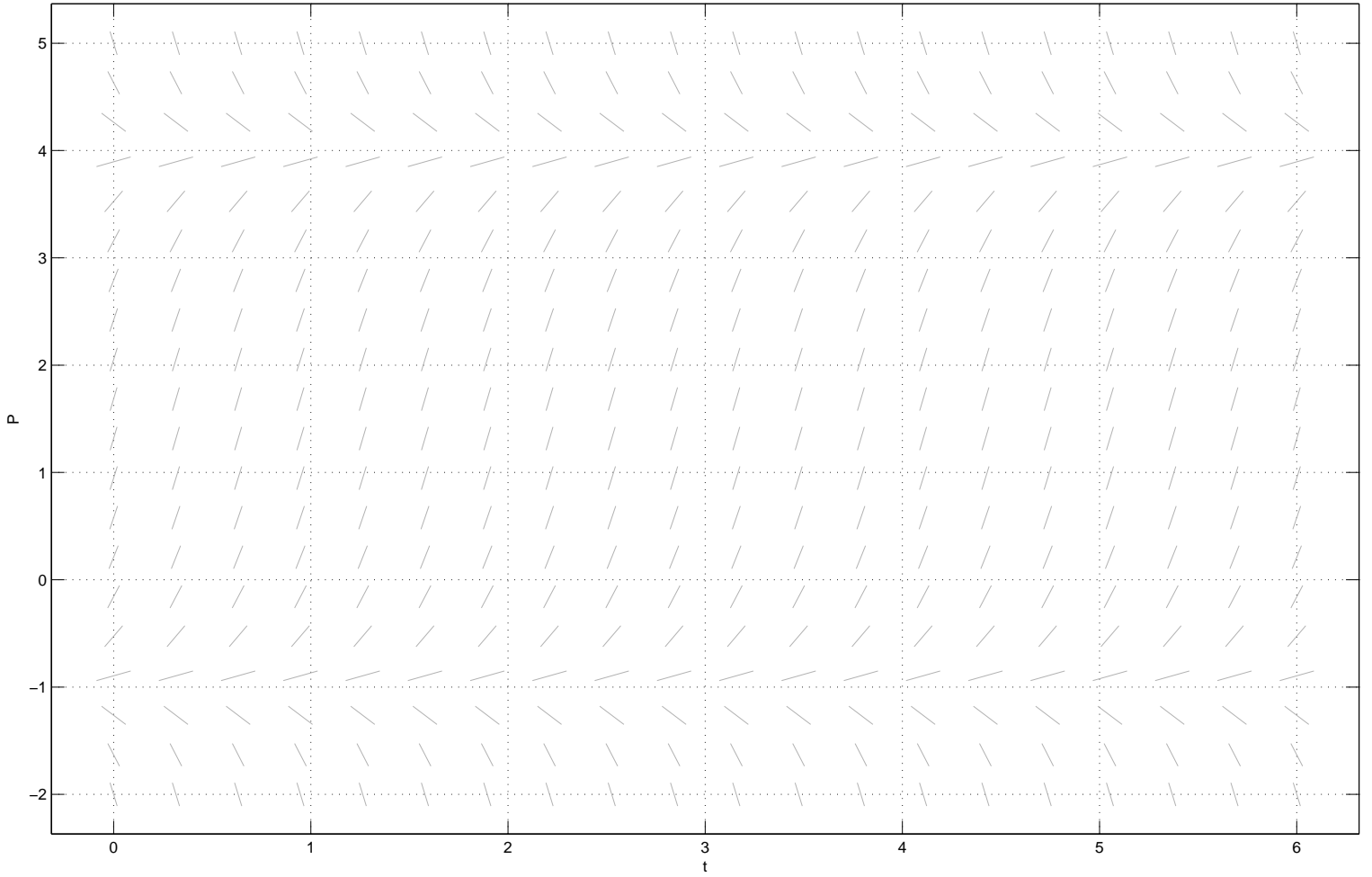
d. The solution with $P(0) = -0.99$ converges to what value of P ?

answer: 4

6. The quantity $I(t)$ of an isotope present satisfies the differential equation $I'(t) = -\alpha I$, where t is in days. If the half-life of the isotope is 20 days, what is α ?

answer: $\alpha = \ln(2)/20 = 0.0347$

$$P' = (1 + P)(4 - P)$$



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