Math 2326, Test I

Name ______________________________

For problems 1-3, solve the differential equations. If you cannot solve for the dependent variable, leave the solution defined implicitly. If no initial condition is given, write the general solution.

1. $\frac{dy}{dt} = \frac{3t^2}{1+3y^2}$, with $y(1) = 1$.

   answer: $y + y^3 = t^3 + 1$

2. $\frac{dy}{dt} = \frac{1+y^3}{3y^2}$, with $y(0) = 0$.

   answer: $y = (e^t - 1)^{1/3}$.

3. $\frac{dy}{dt} - 3y = 2e^{3t}$.

   answer: $y = Ce^{3t} + 2te^{3t}$

4. For the differential equation $y' = t^2 + y^2, y(1) = 2$, take one step using Euler's method with $h = 0.001$, to approximate $y(1.001)$.

   answer: $y(1.001) \approx 2.005$

5. Refer to the direction field plot on the last page to answer the following questions:

   a. (Multiple choice) The differential equation whose field is plotted could be:

      1. $y' = 1 - cos(\pi y)$
      2. $y' = y(4 - y^2)$
      3. $y' = y(2 + y)$
      4. $y' = y(2 + y)^2$
      5. $y' = -1 + cos(\pi y)$
      6. $y' = sin(\pi y)$

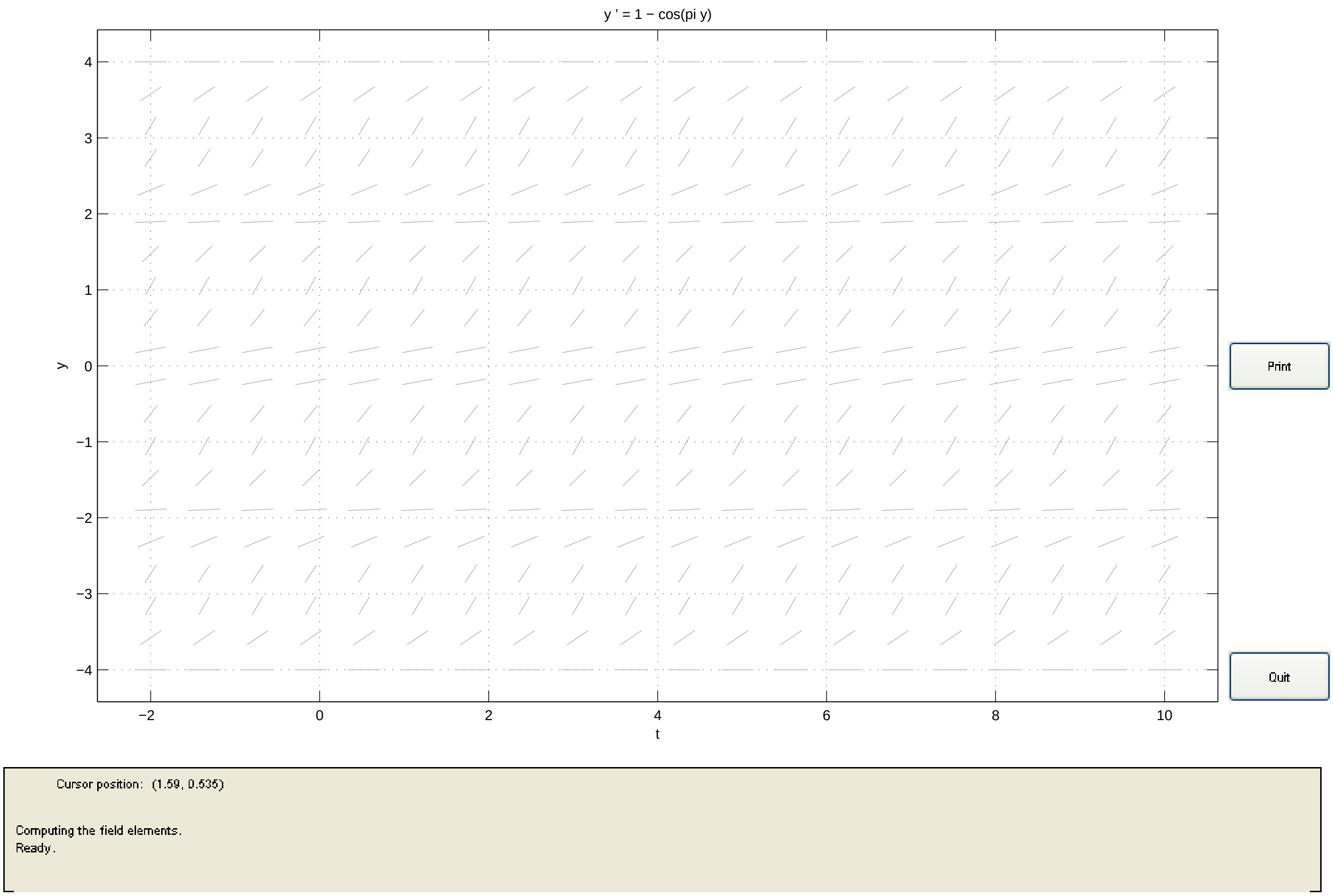
      answer: 1

   b. Classify the equilibrium point at $y = 2$ as a source, sink or node.
      answer: node

   c. The solution to this differential equation with $y(0) = -1.95$ converges to what value of y, as $t \to \infty$?
      answer: 0

6. A 100 gallon tank is full to the brim with pure water, and 10 gallons/minute of a brine solution with 0.5 kg/gallon salt flows into it. Since the tank is full, 10 gallons/minute of well-mixed solution overflows onto the ground. If $S(t)$ is the number of kg of salt in the tank, as a function of time, find and solve a differential equation with boundary conditions for $S(t)$.

   answer: $S'(t) = 0.5(10) - 10\frac{S}{100}$, with $S(0) = 0$
   solution: $S(t) = 50 - 50e^{-0.1t}$


y ' = 1 − cos(pi y)
y
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