

Math 2326, Test II

Name _____

1. a. Find the solution to the following system, with $x(0) = 2, y(0) = 5$.

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 3 & 2 \\ 0 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

answer: $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4e^{3t} - 2e^{-2t} \\ 5e^{-2t} \end{bmatrix}$

- b. Find all equilibrium points of problem 1, and classify each as a source, sink or saddle point.

answer: $(0, 0)$ is a saddle

2. Find the general solution to the system below.

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 0 & 2 \\ -18 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

answer: $\begin{bmatrix} x \\ y \end{bmatrix} = C_1 \begin{bmatrix} \cos(6t) \\ -3\sin(6t) \end{bmatrix} + C_2 \begin{bmatrix} \sin(6t) \\ 3\cos(6t) \end{bmatrix}$

3. a. Write the second order equation $y'' + by' + 9y = 0$ as a system of the form:

$$\begin{bmatrix} y' \\ v' \end{bmatrix} = \begin{bmatrix} \cdot & \cdot \\ \cdot & \cdot \end{bmatrix} \begin{bmatrix} y \\ v \end{bmatrix}$$

answer:

$$\begin{bmatrix} y' \\ v' \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -9 & -b \end{bmatrix} \begin{bmatrix} y \\ v \end{bmatrix}$$

- b. The origin $(0, 0)$ is an equilibrium point for the differential equation system in part (a). For what range of values of b is it a:
- source (answer: $b < 0$)
 - sink (answer: $b > 0$)
 - spiral sink (answer: $0 < b < 6$)

4. The following MATLAB program uses Euler's method to solve what system of differential equations, with what initial conditions? (you don't need to be familiar with MATLAB to figure this out).

```
t = 2;  
x = 2;  
y = -4;  
h = 0.001;  
for i=1:1000  
    f1 = sin(t)*(x^2+y^2);  
    f2 = x/(1+y^2);  
    x = x + h*f1  
    y = y + h*f2  
    t = t + h  
end
```

answer: $x' = \sin(t)(x^2 + y^2)$, $y' = \frac{x}{1+y^2}$, with $x(2) = 2, y(2) = -4$