

Math 2326, Test II

Name _____

1. a. Find the general solution to the following system.

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} -4 & 1 \\ 2 & -3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

answer: $\begin{bmatrix} x \\ y \end{bmatrix} = C_1 e^{-5t} \begin{bmatrix} 1 \\ -1 \end{bmatrix} + C_2 e^{-2t} \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

- b. Find all equilibrium points of problem 1a, and classify each as a source, sink, saddle, spiral source, spiral sink, or center.

answer: $(0, 0)$ is sink

2. a. Find the general solution to the following system.

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ -4 & 6 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

answer: $\begin{bmatrix} x \\ y \end{bmatrix} = C_1 e^{4t} \begin{bmatrix} \cos(2t) \\ \cos(2t) - \sin(2t) \end{bmatrix} + C_2 e^{4t} \begin{bmatrix} \sin(2t) \\ \cos(2t) + \sin(2t) \end{bmatrix}$

- b. Find all equilibrium points of problem 2a, and classify each as a source, sink, saddle, spiral source, spiral sink, or center.

answer: $(0, 0)$ is spiral source

3. a. Solve the following partially decoupled **nonlinear** system:

$$\begin{aligned}x' &= -3t^2xy & x(1) &= 6 \\y' &= 3t^2y^2 & y(1) &= 0.5\end{aligned}$$

answer: $x(t) = 3(3 - t^3), y(t) = \frac{1}{3-t^3}$

- b. Take one step of **Euler's method** to approximate the solution of problem 3a, with $h = 0.1$ (no credit for just evaluating exact solution at $t = 1.1$).

answer: $x(1.1) \approx 5.1, y(1.1) \approx 0.575$

4. Write the second order damped harmonic oscillator equation, $my'' + by' + ky = 0$ in the form (what are $\alpha, \beta, \gamma, \delta$):

$$\begin{bmatrix} y' \\ v' \end{bmatrix} = \begin{bmatrix} \alpha & \beta \\ \gamma & \delta \end{bmatrix} \begin{bmatrix} y \\ v \end{bmatrix}$$

answer:

$$\begin{bmatrix} y' \\ v' \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{k}{m} & -\frac{b}{m} \end{bmatrix} \begin{bmatrix} y \\ v \end{bmatrix}$$