Math 2326, Test II

Name ____________________________

1. a. Find the general solution to the following system.

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} -4 & -2 \\ -1 & -3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

answer: $\begin{bmatrix} x \\ y \end{bmatrix} = C_1 e^{-2t} \begin{bmatrix} 1 \\ -1 \end{bmatrix} + C_2 e^{-5t} \begin{bmatrix} 2 \\ 1 \end{bmatrix}$

b. Give the equations of two lines such that if $(x(0), y(0))$ is on the line, $(x(t), y(t))$ remains on the line for all t.

answer: $y = -x$ and $y = x/2$

c. Find all equilibrium points of problem 1a, and classify each as a source, sink, saddle, spiral source, spiral sink, or center.

answer: $(0, 0)$ is sink

2. Consider the linear system:

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ a & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

a. Find all equilibrium points and classify each as a source sink, saddle, spiral source, spiral sink, or center, if $a > 0$.

answer: $(0, 0)$ is saddle

b. Same question, but now assume $a < 0$.

answer: $(0,0)$ is center

3. a. Find all equilibrium points of the preditor-prey equation:

$$x' = 0.3x - 0.01xy$$
$$y' = 15y(1 - \frac{y}{15}) + 25xy$$

answer: $(0,0), (0,15), (0.6, 30)$

b. What happens to x if $y(0) = 0$? Based on this, does x represent the number of preditors or prey?

answer: $x(t) = Ce^{0.3t} \to \infty$, so x is prey.

c. Take one step of **Euler's method** to approximate the solution of problem 3a, with $h = 0.1$, if $x(0) = 0.6, y(0) = 30$. That is, approximate $x(0.1), y(0.1)$.

answer: $x(0.1) = 0.6, y(0.1) = 30$ (no change)

4. Write the second order damped harmonic oscillator equation, $2y'' + 8y' + 10y = 0$ in the form (what are $\alpha, \beta, \gamma, \delta$?):

$$\begin{bmatrix} y' \\ v' \end{bmatrix} = \begin{bmatrix} \alpha & \beta \\ \gamma & \delta \end{bmatrix} \begin{bmatrix} y \\ v \end{bmatrix}$$

answer:

$$\begin{bmatrix} y' \\ v' \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -5 & -4 \end{bmatrix} \begin{bmatrix} y \\ v \end{bmatrix}$$