

Math 2326, Test II

Name _____

1. a. Find the general solution to the following system, for arbitrary $a \neq 1$.

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & a \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

answer: $\begin{bmatrix} x \\ y \end{bmatrix} = C_1 e^t \begin{bmatrix} 1 \\ 0 \end{bmatrix} + C_2 e^{at} \begin{bmatrix} 1 \\ a - 1 \end{bmatrix}$

- b. For what values of a is $(0, 0)$ a source? (answer: $a > 0$)
c. For what values of a is $(0, 0)$ a sink? (answer: none)
d. For what values of a is $(0, 0)$ a saddle? (answer: $a < 0$)
e. For what values of a are there many equilibrium points? (answer: $a = 0$)

2. Find the solution to the system below with $x(0) = 2, y(0) = 3$.

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 0 & 2 \\ -8 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

answer: $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2\cos(4t) + 1.5\sin(4t) \\ -4\sin(4t) + 3\cos(4t) \end{bmatrix}$

3. a. Write the second order equation $y'' = e^{3t}y^2 + y'$ as a system of the form (Hint: define $v = y'$):

$$\begin{aligned} y' &= f_1(t, y, v) \\ v' &= f_2(t, y, v) \end{aligned}$$

answer: $y' = v, v' = e^{3t}y^2 + v$

- b. The following MATLAB program is to use Euler's method to solve the system of differential equations of problem 3a, with initial conditions $y(0) = 2, y'(0) = 3$. Finish the six incomplete statements. (You don't need to use correct MATLAB syntax).

```
        t = 0;
-->    y = 2
-->    v = 3
        h = 0.001;
        for i=1:1000
-->        f1 = v
-->        f2 = exp(3*t)*y^2 + v
-->        y = y + h*f1
-->        v = v + h*f2
            t = t + h
        end
```

4. Find all equilibrium points of the predator prey system:

$$\begin{aligned}x' &= (0.3 - y/100)x \\ y' &= (15 - y + 25x)y\end{aligned}$$

answer: $(0.6, 30), (0, 15), (0, 0)$