Math 2326, Test II

Name ____________________________

1. a. Find the general solution to the following system.

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 6 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

answer: $\begin{bmatrix} x \\ y \end{bmatrix} = C_1 e^{-3t} \begin{bmatrix} 1 \\ -3 \end{bmatrix} + C_2 e^{2t} \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

b. $(0,0)$ is an equilibrium point; classify it as a source, sink, saddle, spiral source, spiral sink or center.

answer: saddle

c. Are there any initial conditions $(x(0), y(0))$ for which the solution will tend to infinity with time; if so, give one.

answer: anything except a multiple of $(1, -3)$

d. Are there any initial conditions $(x(0), y(0))$ for which the solution will tend to $(0,0)$ with time (other than the equilibrium point $(0,0)$ itself); if so give one.

answer: any nonzero multiple of $(1, -3)$

2. a. Write the second order equation $y'' + by' + ky = 0$ as a system of the form (Hint: define $v = y'$):

$$\begin{bmatrix} y' \\ v' \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} y \\ v \end{bmatrix}$$

answer:

$$\begin{bmatrix} y' \\ v' \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -k & -b \end{bmatrix} \begin{bmatrix} y \\ v \end{bmatrix}$$

b. Give a pair of values for b and k for which the equilibrium point $(0, 0)$ will be a center; then a pair for which it is a spiral sink; then a pair for which it is a (non-spiral) source.

answer: center, $b = 0, k > 0$
spiral sink, $b > 0, k > b^2/4$
source: $b < 0, 0 \leq k \leq b^2/4$

c. The following MATLAB program is to use Euler's method to solve the differential equation of problem 2a (assume b, k are defined), with initial conditions $y(0) = 2, y'(0) = -2$. Finish the six incomplete statements. (You don't need to use correct MATLAB syntax).

```
    t = 0;
--> y = 2;
--> v = -2;
    h = 0.001;
    for i=1:1000
-->    f1 = v;
-->    f2 = -k*y-b*v;
-->    y = y + h*f1;
-->    v = v + h*f2;
       t = t + h
    end
```

3. a. Find all equilibrium points of the system:

$$x' = x(2 - x - y)$$
$$y' = y(3 - 2x - y)$$

answer: $(0,0),(0,3),(2,0),(1,1)$

b. If $x(0) = 0, y(0) = 1$, what does $y(t)$ tend to as $t \to \infty$?. If $x(0) = 1, y(0) = 0$, what does $x(t)$ tend to as $t \to \infty$? Does this appear to model a preditor-prey problem?

answer: If $x = 0$, $y \to 3$; If $y = 0$, $x \to 2$
no, neither species dies out if the other goes extinct.

4. Given that two solutions of

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} -2 & -3 \\ 3 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

are $(e^{-2t}cos(3t), e^{-2t}sin(3t))$ and $(-e^{-2t}sin(3t), e^{-2t}cos(3t))$, find the solution of this system satisfying $x(0) = -3, y(0) = 4$.

answer: $(x,y) = -3e^{-2t}(cos(3t), sin(3t)) + 4e^{-2t}(-sin(3t), cos(3t))$