1. a. Find the general solution to the following system.

\[
\begin{bmatrix}
x' \\
y'
\end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 6 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}
\]

answer: 

\[
\begin{bmatrix} x \\ y \end{bmatrix} = C_1 e^{-3t} \begin{bmatrix} 1 \\ -3 \end{bmatrix} + C_2 e^{2t} \begin{bmatrix} 1 \\ 2 \end{bmatrix}
\]

b. (0, 0) is an equilibrium point; classify it as a source, sink, saddle, spiral source, spiral sink or center.

answer: saddle

c. Are there any initial conditions \((x(0), y(0))\) for which the solution will tend to infinity with time; if so, give one.

answer: anything except a multiple of \((1, -3)\)

d. Are there any initial conditions \((x(0), y(0))\) for which the solution will tend to \((0, 0)\) with time (other than the equilibrium point \((0, 0)\) itself); if so give one.

answer: any nonzero multiple of \((1, -3)\)
2. a. Write the second order equation $y'' + by' + ky = 0$ as a system of the form (Hint: define $v = y'$):

$$
\begin{bmatrix}
  y' \\
  v'
\end{bmatrix} =
\begin{bmatrix}
  A_{11} & A_{12} \\
  A_{21} & A_{22}
\end{bmatrix}
\begin{bmatrix}
  y \\
  v
\end{bmatrix}
$$

answer:

$$
\begin{bmatrix}
  y' \\
  v'
\end{bmatrix} =
\begin{bmatrix}
  0 & 1 \\
  -k & -b
\end{bmatrix}
\begin{bmatrix}
  y \\
  v
\end{bmatrix}
$$

b. Give a pair of values for $b$ and $k$ for which the equilibrium point $(0, 0)$ will be a center; then a pair for which it is a spiral sink; then a pair for which it is a (non-spiral) source.

answer: center, $b = 0, k > 0$
spiral sink, $b > 0, k > \frac{b^2}{4}$
source: $b < 0, 0 \leq k \leq \frac{b^2}{4}$

c. The following MATLAB program is to use Euler’s method to solve the differential equation of problem 2a (assume $b, k$ are defined), with initial conditions $y(0) = 2, y'(0) = -2$. Finish the six incomplete statements. (You don’t need to use correct MATLAB syntax).

```
 t = 0;
 --> y = 2;
 --> v = -2;
 h = 0.001;
 for i=1:1000
   --> f1 = v;
   --> f2 = -k*y-b*v;
   --> y = y + h*f1;
   --> v = v + h*f2;
   t = t + h
 end
```
3. a. Find all equilibrium points of the system:

\[ x' = x(2 - x - y) \]
\[ y' = y(3 - 2x - y) \]

answer: \((0, 0), (0, 3), (2, 0), (1, 1)\)

b. If \(x(0) = 0, y(0) = 1\), what does \(y(t)\) tend to as \(t \to \infty\)? If \(x(0) = 1, y(0) = 0\), what does \(x(t)\) tend to as \(t \to \infty\)? Does this appear to model a predator-prey problem?

answer: If \(x = 0, y \to 3\); If \(y = 0, x \to 2\)
no, neither species dies out if the other goes extinct.

4. Given that two solutions of

\[
\begin{bmatrix}
  x' \\
  y'
\end{bmatrix} =
\begin{bmatrix}
  -2 & -3 \\
  3 & -2
\end{bmatrix}
\begin{bmatrix}
  x \\
  y
\end{bmatrix}
\]

are \((e^{-2t} \cos(3t), e^{-2t} \sin(3t))\) and \((-e^{-2t} \sin(3t), e^{-2t} \cos(3t))\), find the solution of this system satisfying \(x(0) = -3, y(0) = 4\).

answer: \((x, y) = -3e^{-2t}(\cos(3t), \sin(3t)) + 4e^{-2t}(-\sin(3t), \cos(3t))\)