Math 2326, Test II

Name ____________________________

1. a. Find the general solution to the following system.

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 0 & -3 \\ 3 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

answer: $\begin{bmatrix} x \\ y \end{bmatrix} = C_1 \begin{bmatrix} cos(3t) \\ sin(3t) \end{bmatrix} + C_2 \begin{bmatrix} sin(3t) \\ -cos(3t) \end{bmatrix}$

b. $(0,0)$ is an equilibrium point; classify it as a source, sink, saddle, spiral source, spiral sink or center.

answer: center

c. Find the solution if $(x(0), y(0)) = (1, 2)$.

answer: $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} cos(3t) - 2sin(3t) \\ sin(3t) + 2cos(3t) \end{bmatrix}$

2. a. Write the second order equation $y'' - 5y' + 6y = 0$ as a system of the form (Hint: define $v = y'$):

$$\begin{bmatrix} y' \\ v' \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} y \\ v \end{bmatrix}$$

answer:

$$\begin{bmatrix} y' \\ v' \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -6 & 5 \end{bmatrix} \begin{bmatrix} y \\ v \end{bmatrix}$$

b. $(0, 0)$ is an equilibrium point; classify it as a source, sink, saddle, spiral source, spiral sink or center.

answer: source

c. The following MATLAB program is to use Euler's method to solve the differential equation of problem 2a, with initial conditions $y(1) = 3, y'(1) = -4$. Finish the seven incomplete statements. (You don't need to use correct MATLAB syntax).

```
-->  t =  1 ;
-->  y =  3 ;
-->  v = -4 ;
     h = 0.001;
     for i=1:1000
-->     f1 =  v        ;
-->     f2 = -6*y+5*v ;
-->     y = y + h*f1  ;
-->     v = v + h*f2  ;
        t = t + h
     end
```

3. a. Find all equilibrium points of the system:

$$x' = x(12 - x - y)$$
$$y' = y(100 - x^2 - y^2)$$

answer: $(0,0), (0,10), (0,-10), (12,0), (2.258, 9.742), (9.742, 2.258)$

b. If $x(0) = 0, y(0) = -5$, what does $y(t)$ tend to as $t \to \infty$?. answer:

$y(\infty) = -10$

4. Solve the following partially decoupled **nonlinear** system:

$$x' = -3t^2xy \qquad x(1) = 2$$
$$y' = 3t^2y^2 \qquad y(1) = 1$$

answer: $x(t) = 2(2 - t^3), y(t) = 1/(2 - t^3)$