Math 2326, Test II

Name ______________________________

1. a. Find the general solution to the following system.

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 0 & -2 \\ 8 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

answer: $\begin{bmatrix} x \\ y \end{bmatrix} = C_1 \begin{bmatrix} cos(4t) \\ 2sin(4t) \end{bmatrix} + C_2 \begin{bmatrix} -sin(4t) \\ 2cos(4t) \end{bmatrix}$

b. Find all equilibrium points of problem 1a, and classify each as a source, sink, saddle, spiral source, spiral sink, or center.

answer: $(0,0)$ is center

2. a. Find all equilibrium points of the preditor-prey equations:

$$\begin{aligned} x' &= x(14-x) - 0.6xy \\ y' &= -y + 0.2xy \end{aligned}$$

answer: $(0,0), (5,15), (14,0)$

b. What happens to y if $x(0) = 0$? Based on this, does y represent the number of preditors or prey?

answer: $y(t) = Ce^{-t} \to 0$, so y is preditor.

c. Take one step of **Euler's method** to approximate the solution of problem 2a, with $h = 0.1$, if $x(0) = 14, y(0) = 1$. That is, approximate $x(0.1), y(0.1)$.

answer: $x(0.1) = 13.16, y(0.1) = 1.18$

3. Solve the following partially decoupled **nonlinear** system:

$$\begin{aligned} x' &= -t^2xy \qquad & x(1) = 1 \\ y' &= 3t^2y^2 \qquad & y(1) = 1 \end{aligned}$$

answer: $x(t) = (2-t^3)^{1/3}, y(t) = \frac{1}{2-t^3}$

4. Find the solution of

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} -2 & -3 & 4 \\ 0 & 1 & 4 \\ 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

satisfying $x(0) = 3, y(0) = 8, z(0) = 5$, given that the eigenvalues of the matrix are $\lambda_1 = -2, \lambda_2 = 1, \lambda_3 = 3$ with respective eigenvectors $z_1 = (1,0,0), z_2 = (1,-1,0), z_3 = (-2,10,5)$.

answer: $(x, y, z) = 3e^{-2t}(1,0,0) + 2e^t(1,-1,0) + e^{3t}(-2,10,5)$