Math 2326, Test II

Name _____

1. Consider the linear system:

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ a & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

a. Find all equilibrium points and classify each as a source, sink, saddle, spiral source, spiral sink, or center, if $a > 0$.

answer: $(0,0)$ is a saddle

b. Same question, but now assume $a < 0$.

answer: $(0,0)$ is a center

c. If $a = -9$, find the general solution.

answer: $\begin{bmatrix} x \\ y \end{bmatrix} = C_1 \begin{bmatrix} cos(3t) \\ -3sin(3t) \end{bmatrix} + C_2 \begin{bmatrix} sin(3t) \\ 3cos(3t) \end{bmatrix}$

d. Find the solution to problem 1c which satisfies the initial conditions $x(0) = 2, y(0) = 12$.

answer: $\begin{bmatrix} x \\ y \end{bmatrix} = 2 \begin{bmatrix} cos(3t) \\ -3sin(3t) \end{bmatrix} + 4 \begin{bmatrix} sin(3t) \\ 3cos(3t) \end{bmatrix}$

2. Solve the partially decoupled system:

$$x' = x + 1$$
$$y' = xy$$

with $x(0) = 0, y(0) = 1$.

answer: $x(t) = e^t - 1, y(t) = e^{e^t - t - 1}$

3. Find all four equilibrium points of the system:

$$x' = x(5 - x - y)$$
$$y' = y(12 - 2x - 3y)$$

answer: $(0, 0), (0, 4), (5, 0), (3, 2)$

4. Take two Euler method steps to solve problem 2, with $h = 0.01$, to find $x(0.02)$ and $y(0.02)$.

answer: $x(0.02) = 0.0201, y(0.02) = 1.0001$