**Math 2326, Test II**

Name _____

1. Find the general solution to the following system.

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 5 & 2 \\ 1 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

answer: $\begin{bmatrix} x \\ y \end{bmatrix} = C_1 e^{3t} \begin{bmatrix} 1 \\ -1 \end{bmatrix} + C_2 e^{6t} \begin{bmatrix} 2 \\ 1 \end{bmatrix}$

2. Consider the linear system:

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} a & -1 \\ 1 & a \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

   a. Find all equilibrium points and classify each as a source, sink, saddle, spiral source, spiral sink, or center, if $a > 0$.

      answer: $(0, 0)$ is spiral source

   b. Same question, but now assume $a < 0$.

      answer: $(0, 0)$ is spiral sink

   c. Same question, but now assume $a = 0$.

      answer: $(0, 0)$ is center

3. Reduce the second order problem $x'' = x' + 1$ to a system of two first order differential equations, by defining $y = x'$, then find the general solution of the resulting partially decoupled system.

answer: $x(t) = Ce^t - t + D, y(t) = Ce^t - 1$

4. Find all four equilibrium points of the system:

$$x' = (2 - x - y)(3 - x)$$
$$y' = (4 - x^2 - y^2)(4 - y)$$

answer: $(3, 4), (-2, 4), (2, 0), (0, 2)$

5. The following MATLAB program is to use Euler's method to solve the differential equation of problem 1, with initial conditions $x(1) = 2, y(1) = 5$. Finish the seven incomplete statements. (You don't need to use correct MATLAB syntax, as long as the math is correct).

```
-->  t =  1  ;
-->  x =  2  ;
-->  y =  5  ;
     h = 0.001;
     for i=1:1000
-->     f1 =  5*x+2*y ;
-->     f2 =    x+4*y ;
-->     x = x + h*f1  ;
-->     y = y + h*f2  ;
        t = t + h
     end
```