Math 2326, Test II

Name _____

1. a. Find the general solution to the following system.

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} -4 & -2 \\ -1 & -3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

answer: $\begin{bmatrix} x \\ y \end{bmatrix} = C_1 e^{-2t} \begin{bmatrix} 1 \\ -1 \end{bmatrix} + C_2 e^{-5t} \begin{bmatrix} 2 \\ 1 \end{bmatrix}$

b. Find all equilibrium points of problem 1a, and classify each as a source, sink, saddle, spiral source, spiral sink, or center.

answer: $(0, 0)$ is sink

2. a. Find the general solution to the following system.

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ -4 & 6 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

answer: $\begin{bmatrix} x \\ y \end{bmatrix} = C_1 e^{4t} \begin{bmatrix} cos(2t) \\ cos(2t) - sin(2t) \end{bmatrix} + C_2 e^{4t} \begin{bmatrix} sin(2t) \\ cos(2t) + sin(2t) \end{bmatrix}$

b. Find all equilibrium points of problem 2a, and classify each as a source, sink, saddle, spiral source, spiral sink, or center.

answer: $(0,0)$ is spiral source

3. a. Solve the following partially decoupled nonlinear system:

$$\begin{aligned} x' &= x+1 & x(0) &= 0 \\ y' &= xy & y(0) &= e^1 \end{aligned}$$

answer: $x(t) = e^t - 1, y(t) = e^{e^t - t}$

b. Take one step of **Euler's method** to approximate the solution of problem 3a, with $h = 0.1$.

answer: $x(0.1) \approx 0.1, y(0.1) \approx e$

4. Find all equilibrium points of the preditor prey system:

$$\begin{aligned} x' &= (4 - y/10)x \\ y' &= (15 - y + 25x)y \end{aligned}$$

answer: $(1, 40), (0, 15), (0, 0)$