

## Math 2326, Test II

Name \_\_\_\_\_

1. a. Find the general solution to the following system.

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} -4 & -2 \\ -1 & -3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

answer:  $\begin{bmatrix} x \\ y \end{bmatrix} = C_1 e^{-2t} \begin{bmatrix} 1 \\ -1 \end{bmatrix} + C_2 e^{-5t} \begin{bmatrix} 2 \\ 1 \end{bmatrix}$

- b. Find all equilibrium points of problem 1a, and classify each as a source, sink, saddle, spiral source, spiral sink, or center.

answer:  $(0, 0)$  is sink

2. a. Find the general solution to the following system.

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} -2 & -3 \\ 3 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

answer:  $\begin{bmatrix} x \\ y \end{bmatrix} = C_1 e^{-2t} \begin{bmatrix} \cos(3t) \\ \sin(3t) \end{bmatrix} + C_2 e^{-2t} \begin{bmatrix} -\sin(3t) \\ \cos(3t) \end{bmatrix}$

- b. Find all equilibrium points of problem 2a, and classify each as a source, sink, saddle, spiral source, spiral sink, or center.

answer:  $(0, 0)$  is spiral sink

3. a. Solve the following partially decoupled system:

$$\begin{aligned}x' &= y + 2 & x(0) &= 0 \\y' &= y + 1 & y(0) &= 0\end{aligned}$$

answer:  $x(t) = e^t + t - 1, y(t) = e^t - 1$

- b. Take one step of **Euler's method** to approximate the solution of problem 3a, with  $h = 0.1$  (no credit for just evaluating exact solution at  $t = 0.1$ ).

answer:  $x(0.1) = 0.2, y(0.1) = 0.1$

4. a. Find all equilibrium points of the predator-prey equation:

$$\begin{aligned}x' &= 3x - 0.1xy \\y' &= 15y\left(1 - \frac{y}{15}\right) + 5xy\end{aligned}$$

answer:  $(0, 0), (0, 15), (3, 30)$

- b. What happens to  $x$  if  $y(0) = 0$ ? Based on this, does  $x$  represent the number of predators or prey?

answer:  $x(t) = Ce^{3t} \rightarrow \infty$ , so  $x$  is prey.