Math 2326, Test II

Name _____

1. a. Find the solution to the following system, with $x(0) = 2, y(0) = 5$.

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 3 & 2 \\ 0 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

answer: $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4e^{3t} - 2e^{-2t} \\ 5e^{-2t} \end{bmatrix}$

b. Find all equilibrium points of problem 1a, and classify each as a source, sink or saddle point.

answer: $(0, 0)$ is a saddle point

2. Consider the linear system:

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -a & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

a. Classify the equilibrium point (0,0) as a source, sink, saddle, spiral source, spiral sink, or center, if $a > 0.25$.

answer: spiral sink

b. Same question, but now assume $0 < a < 0.25$.

answer: sink

c. Same question, but now assume $a < 0$.

answer: saddle point

3. a. Find all equilibrium points of the preditor-prey equation:

$$x' = 0.3x - 0.01xy$$
$$y' = 15y(1 - \frac{y}{15}) + 25xy$$

answer: $(0, 0), (0, 15), (0.6, 30)$

b. What happens to x if $y(0) = 0$? Based on this, does x represent the number of preditors or prey?

answer: $x(t) = Ce^{0.3t} \to \infty$, so x is prey.

c. Take one step of **Euler's method** to approximate the solution of problem 3a, with $h = 0.1$, if $x(0) = 0.6, y(0) = 30$. That is, approximate $x(0.1), y(0.1)$.

answer: $x(0.1) = 0.6, y(0.1) = 30$ (no change)

4. Solve the partially decoupled system:

$$x' = x$$
$$y' = 2y + x$$

with $x(0) = 1, y(0) = 3$.

answer: $x(t) = e^t, y(t) = 4e^{2t} - e^t$