

Math 2326, Test II

Name _____

1. Find the general solution to the following system.

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ -2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

- a. First, by finding the eigenvalues and eigenvectors of the matrix.
answer: $(x, y) = Ae^{2t}(1, -2) + Be^t(0, 1)$

- b. Now solve as a partially decoupled system.
answer: $x = Ae^{2t}, y = -2Ae^{2t} + Be^t$

- c. Classify the equilibrium point $(0, 0)$ as a source, sink, saddle, spiral source, spiral sink or center. answer: source.

2. Find the solution of the following system, with $x(0) = 2, y(0) = 1$:

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 0 & 3 \\ -12 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

answer: $x = 2\cos(6t) + \frac{1}{2}\sin(6t), y = -4\sin(6t) + \cos(6t)$.

3. a. Find all six equilibrium points of the system:

$$\begin{aligned}x' &= (3 - x - y)(x - 2) \\y' &= (9 - x^2 - y^2)(y - 5)\end{aligned}$$

answer: $(0, 3), (3, 0), (-2, 5), (2, \sqrt{5}), (2, -\sqrt{5}), (2, 5)$

- b. If $x(0) = 2, y(0) = 0$, what does $y(t)$ converge to, as $t \rightarrow \infty$?
answer: $y \rightarrow -\sqrt{5}$

4. The following MATLAB program is to use Euler's method to solve the differential equation of problem 2. Finish the seven incomplete statements:

```
--> t = 0 ;
--> x = 2 ;
--> y = 1 ;
    h = 0.001;
    for i=1:1000
-->     f1 = 3*y      ;
-->     f2 = -12*x   ;
-->     x = x + h*f1 ;
-->     y = y + h*f2 ;
        t = t + h
    end
```