

## Math 2326, Test II

Name \_\_\_\_\_

1. a. Find the general solution to the following system.

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} -5 & 0 \\ 1 & -9 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

answer:  $\begin{bmatrix} x \\ y \end{bmatrix} = C_1 e^{-5t} \begin{bmatrix} 4 \\ 1 \end{bmatrix} + C_2 e^{-9t} \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

- b. (0,0) is an equilibrium point of this system, classify it as a source, sink, saddle, spiral source, spiral sink, or center.

answer: sink

- c. Which of the three direction fields on the last page could correspond to this system?

answer: iii

2. a. Which of the three direction fields on the last page could correspond to the system:

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 1 & -9 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

answer: i

- b. Solve this system as a **partially decoupled system**.

answer:  $x(t) = C_1 e^{5t}, y(t) = C_2 e^{-9t} + C_1/14 e^{5t}$

3. a. Find the general solution to the following system.

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

answer:  $\begin{bmatrix} x \\ y \end{bmatrix} = C_1 \begin{bmatrix} \cos(2t) \\ -\sin(2t) \end{bmatrix} + C_2 \begin{bmatrix} \sin(2t) \\ \cos(2t) \end{bmatrix}$

- b. (0,0) is an equilibrium point, classify it as a source, sink, saddle, spiral source, spiral sink, or center.

answer: center

4. Take one step of **Euler's method**, with  $h = 0.01$ , to approximate  $x(1.01), y(1.01)$ :

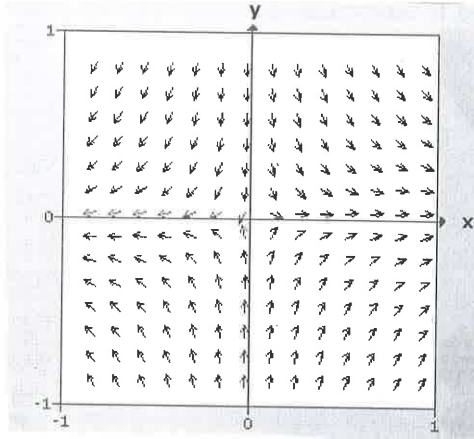
$$\begin{aligned} x' &= -3t^2xy & x(1) &= 2 \\ y' &= 3t^2y^2 & y(1) &= 3 \end{aligned}$$

answer:  $x(1.01) \approx 1.82, y(1.01) \approx 3.27$

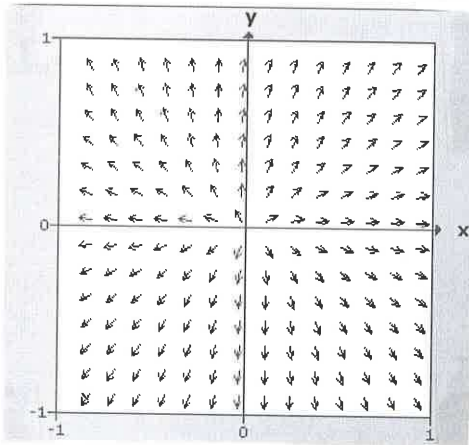
5. Write the second order equation  $y'' + 3y' + 4y = e^t$  as a system of two first order differential equations. Is this system partially decoupled?

answer: no, it is fully coupled

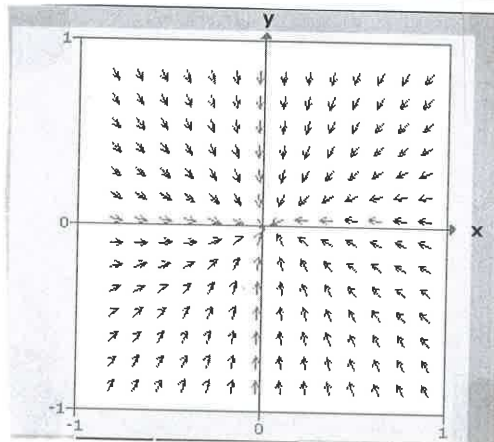
$$\begin{aligned} y' &= v \\ v' &= -3v - 4y + e^t \end{aligned}$$



$i$



$ii$



$iii$