Math 2326, Test II

Name _____

1. Find the solution to the system below with $x(0) = 2, y(0) = 3$.

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 0 & 2 \\ -8 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

answer: $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2cos(4t) + 1.5sin(4t) \\ -4sin(4t) + 3cos(4t) \end{bmatrix}$

2. Consider the linear system:

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} a & 3 \\ -27 & a \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

a. Find all equilibrium points and classify each as a source, sink, saddle, spiral source, spiral sink, or center, if $a > 0$.

answer: $(0,0)$ is spiral source

b. Same question, but now assume $a < 0$.

answer: $(0,0)$ is spiral sink

c. Same question, but now assume $a = 0$.

answer: $(0,0)$ is center

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3. Reduce the second order problem $x'' = x' + e^{2t}$ to a system of two first order differential equations, by defining $y = x'$, then find the general solution of the resulting partially decoupled system.

answer: $x' = y, y' = y + e^{2t}$ solution is $x = Ce^t + D + \frac{1}{2}e^{2t}, y = Ce^t + e^{2t}$

4. Find all four equilibrium points of the system:

$$x' = (3 - x - y)(4 - x)$$
$$y' = (9 - x^2 - y^2)(5 - y)$$

answer: $(4, 5), (-2, 5), (0, 3), (3, 0)$

5. The following MATLAB program is to use Euler's method to solve the differential equation of problem 1. Finish the seven incomplete statements. (You don't need to use correct MATLAB syntax, as long as the math is correct).

```
-->   t = 0    ;
-->   x = 2    ;
-->   y = 3    ;
      h = 0.001;
      for i=1:1000
-->      f1 = 2*y       ;
-->      f2 = -8*x      ;
-->      x = x + h*f1   ;
-->      y = y + h*f2   ;
         t = t + h
      end
```

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