Math 2326, Test III

Name ______________________________

1. Find the general solution:
   $w'' - 5w' + 4w = e^{4t}$

   answer: $w(t) = C_1 e^{4t} + C_2 e^{t} + \frac{1}{3} t e^{4t}$

2. Find any 4 of the 6 equilibrium points of the nonlinear system:

   $\frac{dx}{dt} = -8x^2 - 6xy + 480x$
   $\frac{dy}{dt} = -x^2 y - y^3 + 2500y$

   and classify each as a source, sink, saddle point, spiral source, spiral sink, or center.

   answer: $(0,0)$ is source, $(0,50)$ is saddle, $(0,-50)$ is saddle,
   $(60,0)$ is sink, $(30,40)$ is sink

3. Solve, **using Laplace transforms**
$y'' - 5y' + 4y = e^{-2t}$, with $y(0) = 0, y'(0) = 0$

answer: $y(t) = \frac{1}{18}e^{4t} - \frac{1}{9}e^{t} + \frac{1}{18}e^{-2t}$

4. Find the inverse Laplace transform of $F(s) = \frac{s}{s^2-4s+7}$

answer: $f(t) = e^{2t}cos(\sqrt{3}t) + \frac{2}{\sqrt{3}}e^{2t}sin(\sqrt{3}t)$

5. Find the Laplace transform of the solution to:
$y'' = g(t)$, with $y(0) = 1, y'(0) = 2$, where
$g(t) = 0$ for $t < 2\pi$ and $g(t) = sin(2t)$ for $t \geq 2\pi$.

answer: $L(y) = \frac{s+2}{s^2} + e^{-2\pi s}\frac{2}{s^2(s^2+4)}$