

Math 2326, Test III

Name _____

1. Find the general solution:

$$w'' + 4w' + 4w = 50\sin(t)$$

answer: $w(t) = C_1e^{-2t} + C_2te^{-2t} + 6\sin(t) - 8\cos(t)$

2. Find all 4 of the equilibrium points of the nonlinear system:

$$\begin{aligned}\frac{dx}{dt} &= x(100 - x - 2y) \\ \frac{dy}{dt} &= y(150 - x - 6y)\end{aligned}$$

and classify any two of them a source, sink, saddle point, spiral source, spiral sink, or center.

answer: $(0, 0)$ is source, $(0, 25)$ is saddle, $(100, 0)$ is saddle, $(75, 12.5)$ is sink

3. Find the Laplace transform of the solution to:
 $y'' + 4y' + 4y = 50\sin(t)$, with $y(0) = 2, y'(0) = 3$

answer: $L(y) = \frac{50}{(s^2+1)(s+2)^2} + \frac{2s+11}{(s+2)^2}$

4. Find the inverse Laplace transform of $F(s) = \frac{e^{-3s}}{(s^2-3s+2)}$

answer: $f(t) = u_3(t)[e^{2(t-3)} - e^{t-3}]$

5. Solve **using Laplace transforms**:
 $y'' - 4y = 12$, with $y(0) = 0, y'(0) = 0$

answer: $y(t) = -3 + \frac{3}{2}e^{2t} + \frac{3}{2}e^{-2t}$